

STUDY OF QUEUE WITH WORKING REST PERIOD

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ABSTRACT

To be more precise, a working rest period is any time during the workday that is set aside as non-working time, like daily rest periods or scheduled breaks. Employees are free to take breaks, eat meals, and engage in personal activities during these working rest hours. They are also not obligated to complete job-related tasks. For example, A customer was standing in queue in the bank for availing banking services, but before his turn could have come, bank experienced an unexpected power failure, so, he was forced to wait longer for availing the said banking services. On the other hand, working time describes the times when workers are available to their employer and actively carrying out their responsibilities as defined by applicable national laws or organizational policies. Employees are expected to perform duties allocated to them and fulfill job responsibilities throughout working hours. Employers can guarantee adherence to labor laws concerning maximum working hours, rest breaks, and daily rest times by making a distinction between working time and working rest periods. This distinction gives workers enough time to relax and recover during the workday, protecting their health, safety, and general well-being. For example, A customer was standing in a queue for billing in a grocery store, but before her turn could have come, the printing roll of the printer finished, so, she was forced to wait longer for getting her grocery item billed. In this paper we have discussed about steady state solution operational characteristics and mean number of customers in the system.

KEYWORDS: Queueing Model, Rest period, Customer in system, Working rest period,

INTRODUCTION

Most of the queueing system has been studied in the past about Rest Period” such as P.Vijyalaxmi and et al (2021) has been discussed about the analysis of working vacation



queue with customer impatience and server breakdown which is independent of one another and follow exponential distributions. Customers may become impatient and quit the line during working vacations. To sum up, this research offers methods for analyzing steady-state probability, performance metrics, and cost optimization, and it sheds light on the behavior and performance of queueing systems with different working vacations. A.P Patna and et al (2021) has been studied the review of vacation queueing models in different framework. Vacation queueing systems are thoroughly studied in this work. The research also addresses the difficulties in resolving the issues that vacation queueing systems inherently provide. Shakir Majjid (2019) has been invented the analysis of an M/M/1 queue with working vacation and vacation interruption in which discussion was on least N customers waiting in the system at a service completion moment during the vacation time, the server is programmed to interrupt its vacation and restore normal operation. If not, the server keeps going on vacation until either a vacation ends with the system not empty or a service ends with at least N customers.

Here we will define,

$P_n(t)$ = Probability that there are n-units in the system and the server is busy

$Q_n(x,t)$ = Probability that there are n-units in the system, x time has elapsed in vacation and the server is idle.

The system governs by these difference-differential equations

$$\frac{\partial}{\partial t} Q_0(x,t) + \frac{\partial}{\partial x} Q_0(x,t) = - (a+n(x)) Q_0(t) \tag{1}$$

$$\frac{\partial}{\partial t} Q_n(x,t) + \frac{\partial}{\partial x} Q_n(x,t) = - (a+n(x)) Q_n(t) + a Q_{n-1}(t) \tag{2}$$

$$P'_0(t) = - aP_0 + \int_0^\infty Q_0(x,t) n(x) dx \tag{3}$$

$$P'_n(t) = - (a+\beta) P_n(t) + \int_0^\infty Q_n(x,t) n(x) dx + a P_{n-1}(t) \tag{4}$$

Using the boundary conditions

$$Q_n(0) = \beta P_{n-1}(t) \text{ for } n \geq 0 \tag{5}$$

Let the initial condition be

$$Q(0,0) = 1, Q_n(x,0) = 0, P_n(0) = 0 \text{ for each } x \text{ and } n \tag{6}$$

Defining the probability generating function

$$P(z,t) = \sum_{n=0}^\infty z^n P_n(t) \tag{7}$$

$$Q(x,z,t) = \sum_{n=0}^\infty z^n Q_n(x,t) \tag{8}$$

$$Q(z,t) = \int_0^\infty Q(x,z,t) dx \tag{9}$$

Using (1), (2) and (8)

$$Q(z, x, s) + (s+a+n(x) - az) Q(z, x, s) = 0 \tag{10}$$

which gives

$$\bar{Q}(z, x, s) = \bar{Q}(z, x, s) \exp [-hx \int_0^n n(x) dx] \tag{11}$$

where $h = s + a(1-z)$ and $\bar{Q}(z, x, s)$ is the laplace transform of $Q(x, z, t)$

Similarly, from (3) and (4), we have

$$(s + a + \beta - az - \frac{\beta}{z} \bar{D}(h)) \bar{p}(z, s) = \frac{\beta}{z} \bar{p}_0(s) = \frac{\beta}{z} \bar{p}(s) \bar{D}(h) - \beta \bar{p}_0(s) \tag{12}$$

(5), (10) and (12) give

$$\bar{p}(z, s) = \frac{\beta p_0(s)(z - \bar{D}(h))}{s - az(1-z) + \beta(z - \bar{D}(h))} \tag{13}$$

$$\bar{Q}(z, s) = \frac{1 - \bar{D}(h)}{h} \bar{Q}(z, s) \tag{14}$$

and

$$\bar{Q}(z, 0, s) = \frac{\beta}{z} (\bar{p}(z, s) - P_0) \tag{15}$$

Operational Characteristics

i) Proportion of time the system remains idle state and operative state

from (13) and (14), we will get

$$E1 = \frac{a\beta P_0 E(t)}{\beta(1 - aE(t)) - a} \tag{16}$$

$$E2 = \frac{\beta P_0(1 - aE(t))}{\beta(1 - aE(t)) - a} \tag{17}$$

ii) Mean Number of Customers in the System

$$L_1 = \frac{a^2 \beta P_0 ((\beta - a^2)E(t)^2 + 2E(t))}{2(\beta(1 - aE(t)) - a)^2} \tag{18}$$

$$L_2 = \frac{a \beta P_0 (a^2 E(t)^2 - 2aE(t) + 2)}{2(\beta(1 - aE(t)) - a)^2} \tag{19}$$

$$2 (\beta (1-a E(t)) - a)^2$$

FIGURES AND TABLE In Table - I, value of working vacation queue length or busy period queue length at different arrival rates and service rates. Fig. I shows the behavior of working vacation queue length. It is clear from the graph that if service rate is increases then working vacation queue length is decreases at different batch service. Fig. II shows the behavior of working vacation queue length. It is clear from the graph that if arrival rate λ is increases then queue length is also increases.

| P0 | α | β | a | E(t) | L |
|------|----------|---------|-----|-------|--------|
| 0.06 | 0.2 | 0.1 | 0.2 | 0.125 | 0.0055 |
| 0.06 | 0.2 | 0.1 | 0.3 | 0.125 | 0.0050 |
| 0.06 | 0.2 | 0.1 | 0.4 | 0.125 | 0.0044 |
| 0.06 | 0.2 | 0.1 | 0.5 | 0.125 | 0.0035 |

Table - I

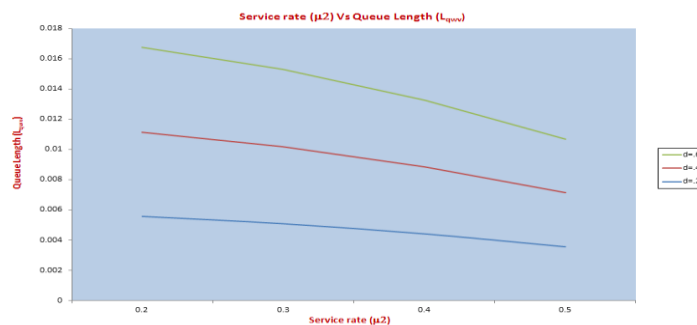


Fig.-I

CONCLUSION: It is clear from the Fig.-I that if service rate is increases then working vacation queue length is decreases at different batch service

REFERENCES

[1] Kumar, A., Application of Markovian process to Queueing with cost Function PhD thesis, 1979, KU, Kurukshetra.

[2] Aggarwal N.N., Some problem in the theory of reliability of queues, PhD Thesis, 1965, K.U.Kurukshetra.

[3] Arora K.L Two server bulk service Queueing process operation Research, Vol. 12, no. 2, 1964 PP 286-294.

- [4] Baba Y., On the $Mx/G/1$ Queue with vacation time operation, Research letters vol. 5, No. 2, 1986, PP 93-98.
- [5] Tuteja, R.K., Solution of transient state limited space queuing problem with arrival departure rates depending on queue length metrika 17, 1971, pp. 207-214.
- [6] Tijmsh H. Analysis of inventory models, mathematical center tracks 40, 1972.
- [7] Tamrakar, G. K. (2023). Studies on some infinite buffer batch size dependent bulk service queues with queue size dependent vacation (Doctoral dissertation, IIT (BHU) Varanasi).
- [8] Tamrakar, G. K., & Banerjee, A. (2023). On Steady State Analysis of an Infinite Capacity $M^x/G^{(a, \gamma)}/1$ Queue with Optional Service and Queue Length Dependent Single (Multiple) Vacation. *Queueing Models and Service Management*, 6(1), 27-61.
- [9] Goswami, C., & Selvaraju, N. (2013). A working vacation queue with priority customers and vacation interruptions. *International Journal of Operational Research*, 17(3), 311-332.
- [10] Divya, K., & Indhira, K. (2024). literature survey on queueing model with working vacation. *reliability: theory & applications*, 19(1 (77)), 40-49.
- [11] Gao, S., Wang, J., & Li, W. W. (2014). An $M/G/1$ retrial queue with general retrial times, working vacations and vacation interruption. *Asia-Pacific Journal of Operational Research*, 31(02), 1440006.
- [12] Upadhyaya, S. (2016). Queueing systems with vacation: an overview. *International journal of mathematics in operational research*, 9(2), 167-213.
- [13] Laxmi, P. V., Rajesh, P., & Kassahun, T. W. (2021). Analysis of a variant working vacation queue with customer impatience and server breakdowns. *International Journal of Operational Research*, 40(4), 437-459.
- [14] Bouchentouf, A. A., Boualem, M., Yahiaoui, L., & Ahmad, H. (2022). A multi-station unreliable machine model with working vacation policy and customers' impatience. *Quality Technology & Quantitative Management*, 19(6), 766-796.
- [15] Panta, A. P., Ghimire, R. P., Panthi, D., & Pant, S. R. (2021). A Review of Vacation Queueing Models in Different Framework. *Ann. Pure Appl. Math*, 24, 99-121.



[16] Majid, S., & Manoharan, P. (2019). Analysis of an M/M/1 queue with working vacation and vacation interruption. *Applications and Applied Mathematics: An International Journal (AAM)*, 14(1), 2.