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ISSN 2348 - 7550

SEMI SIMPLE TERNARY IDEALS IN TERNARY SEMIRINGS

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Abstract

We introduce the belief of generalized semi-best in a ternary semiring. Various examples to set up a courting among beliefs, bi-beliefs, quasi-beliefs and generalized semi-beliefs are furnished. A criterion for a commutative ternary semiring with none divisors of 0 to a ternary department semiring is given.

Keywords and phrases: Ternary semiring, generalized semi-ideals and ideals in ternary semirings, ternary division semiring

1. Introduction

Ternary earrings and their systems have been investigated through Lister [4] in 1971. In fact, Lister characterised the ones additive subgroups of earrings which might be closed beneathneath the triple product. In 2003, T. K. Dutta and S. Kar [3] brought the belief of a ternary semiring as a generalization of a ternary ring. A ternary semiring arises evidently as follows. Consider the subset Z^- of all poor integers of Z. Then $Z^-$is an additive semigroup that's closed beneathneath the triple product. Z^- is a ternary semiring. Note that Z^- does now no longer shape a semiring. In [3] T. K. Dutta and S. Kar brought the notions of left/right/lateral beliefs of ternary semirings and additionally characterised everyday ternary semirings. In 2005, S. Kar [1] brought the notions of quasi-beliefs and bi-beliefs in a ternary semiring. The belief of a generalized semi-best in a hoop has been brought and studied through T. K. Dutta in [2]. In this paper we introduce the belief of a generalized semi- best in a ternary semiring and look at them. Also, we set up a dating among generalized semi-beliefs, beliefs, bi-beliefs, etc. in a ternary semiring to look at a few residences of a generalized semi-beliefs in ternary semirings.

2. Preliminaries

For preliminaries we refer to [1] and [3].

Definition 2.1. An additive commutative semigroup S, together with a ternary multiplication denoted by [ ] is said to be a ternary semiring if $[abc]de] = [a[bcd]e] = [ab[cde]]$,

i) $[(a + b)cd] = [acd] + [bcd]$,

ii) $[a(b + c)d] = [abd] + [acd]$,

iii) $[ab(c + d)] = [abc] + [abd]$ for all $a, b, c, d, e \in S$.

Throughout, S will denote a ternary semiring unless otherwise stated.

Definition 2.2. If there exists an element $0 \in S$ such that $0 + x = x$ and $[0xy] = [xy0] = [x0y] = 0$ for all $x, y \in S$, then 0 is called the zero element of S. In this case we say that S is a ternary semiring with zero.

Definition 2.3. S is called a commutative ternary semiring if $[abc] = [bac] = [bca]$, for all $a, b, c \in S$.

Definition 2.4. An additive subsemigroup T of S is called a ternary subsemir- ing of S if $[t_1t_2t_3] \in T$ for all $t_1, t_2, t_3 \in T$.

Definition 2.5. An element a in S is called regular if there exists an element $x \in S$ such that $[axa] = a$. S is called regular if all of its elements are regular.

Definition 2.6. S is said to be zero-divisor free (ZDF) if for $a, b, c \in S$, $[abc] = 0$ implies that $a = 0$ or $b = 0$ or $c = 0$.

Definition 2.7. S with $|S| \geq 2$ is called a ternary division semiring if for any non-zero element a of S , there exists a non-zero element $b \in S$ such that $[abx] = [bax] = [xab] = [xba] = x$, for all $x \in S$.

Definition 2.8. A left (right/lateral) ideal I of S is an additive subsemigroup of S such that $[s_1s_2i] \in I$ ($[is_1s_2] \in I$/$[s_1is_2] \in I$) for all $i \in I$, for all $s_1, s_2 \in S$. If I is a left, a right and a lateral ideal of S, then I is called an ideal of S.

Definition 2.9. An additive subsemigroup Q of S is called a quasi-ideal of S if $[QSS] \cap ([SQS] + [SSQSS]) \cap [SSQ] \subseteq Q$.

Definition 2.10. A ternary subsemiring B of S is called a bi-ideal of S if $[BSBSB] \subseteq B$.

3. Generalized semi-ideals in ternary semirings

Generalized semi-ideals in semirings are introduced and studied by T .K. Dutta in [1]. As a generalization, we define generalized semi-ideals in ternary semirings.

Definition 3.1. A non-empty subset A of S satisfying the condition $a + b \in A$, for all $a, b \in A$

is called

i) generalized left semi-ideal of S if $[[xxx]xa] \in A$ for all $a \in A$ for all $x \in S$,

ii) generalized right semi-ideal of S if $[axx]xx] \in A$ for all $a \in A$, for all $x \in S$,

iii) generalized lateral semi-ideal of S if $[xxa]xx] \in A$ for all $a \in A$, for all $x \in S$,

iv) generalized semi-ideal of S if it is a generalized left semi-ideal, a generalized right semi-ideal and a generalized lateral semi-ideal of S.

Example 3.2. Consider a ternary semiring Z of all integers. The subset A of Z containing all non-negative integers and the set B of all non-positive integers are generalized semi-ideals of Z.

Remark 3.3. The ideas of generalized semi-perfect and ternary subsemiring are unbiased in S. This manner this is each ternary subsemiring of S want now no longer be a generalized semi-perfect of S and each generalized semi-perfect of S want now no longer be a ternary subsemiring of S. For this, don't forget the subsequent examples.

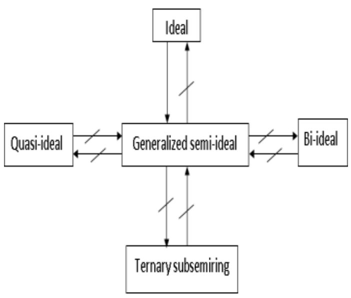
Example 3.4. Let $S = M_2\,(Z_0^-)$ be the ternary semiring of the set of all 2×2 square matrices over Z_0 , the set of all non-positive integers.

Let $T = \{ \quad a \quad 0 \quad / a \in Z^- \}$. T is a ternary subsemiring of S, but it is not a generalized semi-ideal of S.

Example 3.5. Let $S = . . . , 2i, i, 0, i, 2i, . . .$ be a ternary semiring with respect to addition and complex triple multiplication. Let $A = 0, i, 2i,$ A is a generalized semi-ideal of S, but not a ternary subsemiring of S.

Every ideal of S is a generalized semi-ideal of S but the converse need not be true.

Example 3.6. Every quasi-best want now no longer be a generalized semi-best and each generalized semi-best want now no longer be a quasi-best of S. In Example 3.4), T is a quasi-best of S, however it isn't always a generalized semi-best of S. In Example 3.5, A is a



generalized semi-best of S, however now no longer a quasi-best of S. Every quasi-best is a bi-best in S [2]. Hence, the bi-beliefs and generalized semi-beliefs in S are unbiased concepts. The flow-chart of the connection among the beliefs, bi-beliefs, quasi-beliefs, ternary subsemiring and generalized semi-beliefs in a ternary semiring is given below.

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