ijates
ISSN 2348 - 7550

Decision Making Problems and Pattern Recognition in New Fuzzy Divergence Model

Amit Tuteja, Ashwani Sethi

Guru Kashi University, Talwandi Sabo

Abstract

It has been demonstrated that in practical scenarios, a single mathematical model calculating distance in probability space is ineffective. This passion motivates us to start a new divergence model in order to persuade elasticity in a variety of areas. One of the research topics aimed at application fields is the thorough learning of scrupulous features for the legitimacy of distance models in a fuzzy environment. The primary goal of this study is to make progress in this direction. We have used the newly developed distance model to the disciplines of problem solving and pattern recognition. We created a numerical image to explain the anticipated solution approach.

Keywords: *Probability spaces, Fuzzy entropy, Distance model, Decision making problems, Pattern recognition.*

1. Introduction

The distance models in probability spaces contribute imperative responsibility because of their applications towards multiplicity of disciplines and one of the nucleus concerns is to find such a suitable distance model. Such a divergence model is a constructive instrument in solving countless optimization problems pertaining to a multiplicity of disciplines. It has been pragmatic that the largely imperative and constructive divergence model in probability spaces is payable to Kullback and Leibler [12]. With the maintenance view of the deep-seated properties and application areas of such divergence models in probability spaces, many new divergence models have been created by an assortment of researchers. Some of these the fundamental divergence models are payable to Havrda-Charvat's [8], Renyi's [16] etc. Dragomir [6] has created numerous innovative classes of universal divergence models, offered their foremost properties and produced the association with well recognized information models like Csiszar's $f-$divergence or the Jeffreys divergence [9].

The peculiarity that the quantitative information models wrought by an assortment of researchers have awfully pleasing properties and provides marvelous applications in a mixture of disciplines of mathematics forced the researchers to ascertain its shortcoming that it takes into description merely the probabilities coupled with the events and not their consequence or magnitude. But, in countless authentic life situations it becomes the inevitability to take into account both the equally imperative aspects, that is, magnitude as well as eminence. Stimulated by this idea, it was Guiasu [7] who tailored the concept of weighted information. With this convincing idea Kapur [11] created several largely conceivable properties of a suitable weighted divergence model and subsequently developed the following advantageous divergence models:

$$D_1(P;Q:W)=\sum_{i=1}^{n}w_i\left[p_i\log\frac{p_i}{q_i}-p_i+q_i\right]$$

(1.1)

$$D_2(P;Q:W)=\sum_{i=1}^{n}w_i\left[\frac{p_i-p_i In\,p_i-q_i+q_i In\,q_i}{In\,q_i}+p_i-q_i\right]$$

(1.2)

On the other hand, Zadeh's [19] incredible theory of fuzzy sets competent with vague concepts extended the scope of research with pervasive applications in numerous areas. This innovation gave confinement to divergence models in fuzzy environment which provide extraordinary magnitude because of their extensive utilization in large numbers of fields. Bhandari and Pal [3] recommended an innovative divergence model for such situations. For the fuzzy distributions, De Luca and Termini [5] wrought fuzzy entropic model whereas Bhandari and Pal [3] and Couso et al. [4] twisted fuzzy divergence models to undertake the widespread situations. Parkash [14] twisted a new divergence measure for fuzzy distributions whereas Parkash et al. [15] projected several new models for fuzzy distributions and deliberated their comprehensive properties for their legitimacy. Some recent developments regarding the divergence models in probability spaces have been made by Kumari and

Sharma [13], Agahi [1], Torra et al.[18], Anastassiou [2] etc. whereas certain investigations of these models for fuzzy distributions have wrought by Joshi and Kumar [10], Saraswat and Khatod [17] etc.

This is reiterated that one of the application ranges of research interest is decision making while another application area of interest is pattern recognition which provides the study of machines monitoring the surroundings. These days such studies are made by machine intelligence providing upcoming area of research. Within the area of machine learning, pattern recognition circumscribe an ample area of information processing problems of enormous practical implications. Alternatively, it can be depicted as taking in raw data and performing an action based on the confederation of data and consequently provides collection of methods for supervised learning.

2. A New Divergence Measure For Fuzzy Set

With fuzzy values $\zeta_{\breve{P}}(\breve{x}_i)$ and $\zeta_{\breve{Q}}(\breve{x}_i)$ available from fuzzy sets $\breve{P}$ and $\breve{Q}$, we at this instant outline a new fuzzy divergence model given by

$$\breve{L}(\breve{P},\breve{Q})=\sum_{i=1}^{n}\left[\begin{array}{l}\left(\zeta_{\breve{P}}(\breve{x}_i)+\zeta_{\breve{Q}}(\breve{x}_i)\right)\ln\left(\dfrac{\zeta_{\breve{P}}(\breve{x}_i)+\zeta_{\breve{Q}}(\breve{x}_i)}{\frac{2}{3}\left\{\zeta_{\breve{P}}(\breve{x}_i)+\sqrt{\zeta_{\breve{P}}(\breve{x}_i)\zeta_{\breve{Q}}(\breve{x}_i)}+\zeta_{\breve{Q}}(\breve{x}_i)\right\}}\right)\\ +\left(2-\zeta_{\breve{P}}(\breve{x}_i)-\zeta_{\breve{Q}}(\breve{x}_i)\right)\ln\left(\dfrac{2-\zeta_{\breve{P}}(\breve{x}_i)-\zeta_{\breve{Q}}(\breve{x}_i)}{\frac{2}{3}\left\{2-\zeta_{\breve{P}}(\breve{x}_i)-\zeta_{\breve{Q}}(\breve{x}_i)+\sqrt{\left(1-\zeta_{\breve{P}}(\breve{x}_i)\right)\left(1-\zeta_{\breve{Q}}(\breve{x}_i)\right)}\right\}}\right)\end{array}\right]$$

(2.1)

Theorem: $\breve{L}(\breve{P},\breve{Q})$ is an authentic divergence measure under fuzzy environment.

Proof: Equation (2.1), shapes the perception of the following:

(i) $\breve{L}(\breve{P},\breve{Q})=\breve{L}(\breve{Q},\breve{P})$;

(ii) $\breve{L}(\breve{P},\breve{Q})=0$ iff $\zeta_{\breve{P}}(\breve{x}_i)=\zeta_{\breve{Q}}(\breve{x}_i)$

(iii) We organize the division of the set $\breve{X}$ when $\breve{x}\in\breve{X}$ in six subsets.

In set $\breve{X}_1$, $\breve{P}\cup\breve{R} \Leftrightarrow \zeta_{\breve{P}\cup\breve{R}}(\breve{x}) = \zeta_{\breve{R}}(\breve{x})$; $\breve{P}\cap\breve{R} \Leftrightarrow \zeta_{\breve{P}\cap\breve{R}}(x) = \zeta_{\breve{P}}(\breve{x})$ etc.

Now, $\breve{L}\left(\breve{P}\cup\breve{R},\breve{Q}\cup\breve{R}\right)=\breve{L}\left(\breve{R},\breve{R}\right)=0$. Similarly, one can prove that $\breve{L}\left(\breve{P}\cap\breve{R},\breve{Q}\cap\breve{R}\right)=\breve{L}\left(\breve{P},\breve{Q}\right)$

Hence, $L\left(\breve{P},\breve{Q}\right)$ is an authentic fuzzy divergence model.

3. Significant Properties of Divergence Model $K\left(\tilde{P},\tilde{Q}\right)$

Here, we establish the largely imperative and significant properties of the divergence model in the form of following theorem after separating $\breve{X}$ into $\breve{X}_1$ and $\breve{X}_2$.

Theorem 3.1. (a) $\breve{L}\left(\breve{P},\overline{\breve{P}}\right)=\breve{L}\left(\overline{\breve{P}},\breve{P}\right)$.

(b) $\breve{L}\left(\overline{\breve{P}},\overline{\breve{Q}}\right)=\breve{L}\left(\breve{P},\breve{Q}\right)$.

(c) $\breve{L}\left(\breve{P},\overline{\breve{Q}}\right)=\breve{L}\left(\overline{\breve{P}},\breve{Q}\right)$.

(d) $\breve{L}\left(\breve{P},\breve{Q}\right)+\breve{L}\left(\overline{\breve{P}},\breve{Q}\right)=\breve{L}\left(\overline{\breve{P}},\overline{\breve{Q}}\right)+\breve{L}\left(\breve{P},\overline{\breve{Q}}\right)$.

Proof: Using equation (2.1), we get

$$\breve{L}\left(\breve{P},\overline{\breve{P}}\right)=\sum_{i=1}^{n}2\ln\left(\frac{1}{\frac{2}{3}\left\{1+\sqrt{\zeta_{\breve{P}}(\breve{x}_i)\left(1-\zeta_{\breve{P}}(\breve{x}_i)\right)}\right\}}\right)$$

Again from equation (2.1), we get

$$\breve{L}\left(\overline{\breve{P}},\breve{P}\right)=\sum_{i=1}^{n}2\ln\left(\frac{1}{\frac{2}{3}\left\{1+\sqrt{\zeta_{\breve{P}}(\breve{x}_i)\left(1-\zeta_{\breve{P}}(\breve{x}_i)\right)}\right\}}\right)=\breve{L}\left(\breve{P},\overline{\breve{P}}\right)$$ which proves (a).

(b) Proceeding as above, we can prove this part also.

(c) Proceeding as above, we can prove this part also.

(d) Using (b) and (c), this part is proved.

4. Applications of Divergence Measure For Fuzzy Sets to Decision Making Problems and Pattern Recognition

(a) Multi-criteria Decision Making Problem

Let $\breve{O}=\{\breve{O}_1, \breve{O}_2,...,\breve{O}_m\}$ be a set of choices, $\breve{S}=\{\breve{S}_1, \breve{S}_2,...,\breve{S}_n\}$ be a set of precedent. With the description of choices $\breve{O}_{\tilde{i}}$ in terms of precedent $\breve{S}$ are expressed as following fuzzy sets: $\breve{O}_{\tilde{i}}=\left\{\left\langle \breve{S}_{\tilde{j}}, \zeta_{\tilde{i}\tilde{j}}\right\rangle \mid \breve{S}_{\tilde{j}} \in \breve{S}\right\}$, we suggest the following algorithm:

1. Solve the (+ ve)-ideal solution $\breve{O}^{+}$ and (-ve)-ideal solution $\breve{O}^{-}$:

$$\breve{O}^{+}=\{\langle\zeta_{1+}\rangle, \langle\zeta_{2+}\rangle,...,\langle\zeta_{n+}\rangle\},$$

$$\breve{O}^{-}=\{\langle\zeta_{1-}\rangle, \langle\zeta_{2-}\rangle,...,\langle\zeta_{n-}\rangle\},$$

$$\left.\begin{array}{l}\left\langle\zeta_{\tilde{j}+}\right\rangle=\left\langle\max\limits_{\tilde{i}} \zeta_{\tilde{i}\tilde{j}}\right\rangle \\ \left\langle\zeta_{\tilde{j}-}\right\rangle=\left\langle\min\limits_{\tilde{i}} \zeta_{\tilde{i}\tilde{j}}\right\rangle\end{array}\right\}$$

(4.1)

2. Using (2.1) find $\breve{L}\left(\breve{O}^{+};\breve{O}_{\tilde{i}}\right)$ and $\breve{L}\left(\breve{O}^{-};\breve{O}_{\tilde{i}}\right)$.

3. Solve the relative divergence for fuzzy set $\breve{L}\left(\breve{O}_{i}\right)$ of option $\breve{O}_{i}$ w.r.t to $\breve{O}^{+}$ and $\breve{O}^{-}$, where

$$\breve{L}\left(\breve{O}_{\tilde{i}}\right)=\frac{\breve{L}\left(\breve{O}^{+};\breve{O}_{\tilde{i}}\right)}{\breve{L}\left(\breve{O}^{+};\breve{O}_{\tilde{i}}\right)+\breve{L}\left(\breve{O}^{-};\breve{O}_{\tilde{i}}\right)}$$

(4.2)

4. On the basis of the comparative fuzzy divergence, grade the preference of all alternatives.

5. Assign most appropriate option $\breve{O}_{\tilde{k}}$ with the smallest $\breve{L}\left(\breve{O}_{\tilde{k}}\right)$.

5. A Numerical Example

Assume that a venture firm desires investment of assured quantity of money in the finest choice from the five choices: A software firm $\breve{O}_1$, a pharmaceutical firm $\breve{O}_2$, a textile firm $\breve{O}_3$, an automobile firm $\breve{O}_4$ and an air conditioner firm $\breve{O}_5$. The venture firm wants to take a judgment according to the following four precedents: (1) H_1; investigation of risk. (2) H_2; analysis of growth. (3) H_3; investigation of social-political impact (4) H_4; investigation of atmospheric impact. The five possible options $\breve{O}_i$ are concealed by four precedents. Then through step 1, we have solveed (+ve)-ideal solution $\breve{O}^+$ and the (-ve)-ideal solution $\breve{O}^-$. Finally, through step 2, we obtain the following tables:

Table-I: $\breve{L}\left(\breve{O}^+;\breve{O}_i\right)$

$\breve{L}\left(\breve{O}^+;\breve{O}_1\right)$	0.1807
$\breve{L}\left(\breve{O}^+;\breve{O}_2\right)$	0.0849
$\breve{L}\left(\breve{O}^+;\breve{O}_3\right)$	0.1871
$\breve{L}\left(\breve{O}^+;\breve{O}_4\right)$	0.1738
$\breve{L}\left(\breve{O}^+;\breve{O}_5\right)$	0.4106

Table-II: $\breve{L}\left(\breve{O}^-;\breve{O}_i\right)$

$\breve{L}\left(\breve{O}^-;\breve{O}_1\right)$	1.1411
$\breve{L}\left(\breve{O}^-;\breve{O}_2\right)$	1.1526
$\breve{L}\left(\breve{O}^-;\breve{O}_3\right)$	0.8936
$\breve{L}\left(\breve{O}^-;\breve{O}_4\right)$	0.9961
$\breve{L}\left(\breve{O}^-;\breve{O}_5\right)$	0.9958

Step 3: Using equation (4.2), calculate $\breve{L}\left(\breve{O}_i\right)$ as follows:

Table-III: $\breve{L}\left(\breve{O}_i\right)$

$\breve{L}\left(\breve{O}_1\right)$	0.1367
$\breve{L}\left(\breve{O}_2\right)$	0.0686
$\breve{L}\left(\breve{O}_3\right)$	0.1732

$\breve{L}(\breve{O}_4)$	0.1486
$\breve{L}(\breve{O}_5)$	0.2919

Step 4: Order of ranking for five choices is:

$$O_2 \succ O_1 \succ O_4 \succ O_3 \succ O_5 .$$

Step 5: From the above step, O_2 is the reasonable choice.

(b) Pattern Recognition

In sequence to exhibit the applications of our model in this field, suppose we are given r known patterns $E_1, E_2, E_3, ..., E_r$ revealed by subsequent fuzzy sets framing reference X with classifications $S_1, S_2, S_3, ..., S_r$.

$$E_{\tilde{i}} = \left\{ \left\langle \hat{x}_l, \zeta_{E_{\tilde{i}}}(\hat{x}_l) \right\rangle \middle/ \hat{x}_l \in X \right\},$$

Given an inconsonant pattern Q, exhibited by

$$\hat{Q} = \left\{ \left\langle \hat{x}_l, \zeta_{\hat{Q}_i}(\hat{x}_l) \right\rangle \middle/ \hat{x}_l \in X \right\}.$$

Our aspiration is to regiment Q to one of the classes $S_1, S_2, S_3, ..., S_r$. We define the process of accredit Q to S_{g^*} as

$$g^* = \arg\min_g \left\{ L\left(E_l, \hat{Q}\right) \right\}.$$

According to the algorithm defined above, the obsessed pattern can be perceived and most appropriate class can be tabbed.

An Illustrative Example

Here we have given three known patterns E_1, E_2 and E_3 having classifications S_1, S_2 and S_3 and revealed by the following sets:

$$E_1 = \{\langle x_1, 0.8\rangle, \langle x_2, 0.3\rangle, \langle x_3, 0.2\rangle\}$$
$$E_2 = \{\langle x_1, 0.5\rangle, \langle x_2, 0.1\rangle, \langle x_3, 0.6\rangle\}$$
$$E_3 = \{\langle x_1, 0.4\rangle, \langle x_2, 0.9\rangle, \langle x_3, 0.7\rangle\}$$

Provided that we have an inconsonant pattern $\hat{Q}$, revealed by the fuzzy set

$\hat{Q} = \{\langle x_1, 0.3\rangle, \langle x_2, 0.5\rangle, \langle x_3, 0.8\rangle\}$.

Our aspiration is to regiment Q be classified among S_1, S_2 and S_3. From (3.1), we can enumerate fuzzy divergence value of $\hat{L}(E_m, Q)$, $m = \{1, 2, 3\}$.

$\hat{L}(E_1, \hat{Q}) = 0.3534$; $\hat{L}(E_2, \hat{Q}) = 0.1490$; $\hat{L}(E_3, \hat{Q}) = 0.1924$.

Hence, it is perceived that $\hat{Q}$ has been restricted to S_2 correctly.

Concluding Remarks

In the active literature of divergence models, one gets enthused to outline innovative models as to persuade suppleness in dissimilar disciplines. A new-fangled fuzzy divergence model with advantageous properties has been investigated in the present paper. On the basis of proposed measure, an algorithm to negotiate with decision making problems and pattern recognition has been illustrated. Numerical design has been provided to justify our claim. Such new fuzzy divergence models can be shaped for furtherance of research in these disciplines.8

References

[1] Agahi, H. (2019). A modified Kullback–Leibler divergence for non-additive measures based on Choquet integral. *Fuzzy Sets and Systems*, 367, 107–117.

[2] Anastassiou, G.A. (2020). *“Fractional Conformable Approximation of Csiszar’s* f-*Divergence”,* In: Intelligent Analysis: Fractional Inequalities and Approximations Expanded. Studies in Computational Intelligence, 886, 463-480 Springer, Cham.

[3] Bhandari, D., Pal, N.R. (1993). Some new information measure for fuzzy sets. *Information Science*, 67(3), 209-228.

[4] Couso, I., Jains, V., Montes, S. (2000). Fuzzy divergence measures. *Acta Univ. M. Belii. Ser. Math.*, 8, 21-26.

[5] De Luca, A., Termini, S. (1972). A definition of non-probabilistic entropy in the setting of fuzzy set theory. *Information and Control*, 20(4), 301-312.

[6] Dragomir, S. S. (2005). Some general divergence measures for probability distribution. *Acta Mathematica. Hungarica*, 109,331-345.

[7] Guiasu, S. (1971). Weighted entropy. *Reports on Mathematical Physics*, 2, 165-171.

[8] Havrda, J. H. and Charvat, F. (1967). Quantification methods of classification process: Concept of structural α-entropy. *Kybernetika*, 3, 30-35.

[9] Jeffreys, H. (1946). An Invariant Form for the Prior Probability in Estimation Problems. *Proc. of the Royal Society of London. Series A*, 186, 453-461.

[10] Joshi, R. and Kumar, S. (2018). An (R',S')-norm fuzzy relative information measure and its applications in strategic decision-making. *Comput. Appl. Math.*, 37(4), 4518–4543.

[11] Kapur, J.N. (1995). Measures of Information and their Applications, Wiley Eastern, New York.

[12] Kullback, S. and Leibler, R.A. (1951). On information and sufficiency. *The Annals Mathematical Statistics*, 22(1), 79–86.

[13] Kumari, R. and Sharma, D. K. (2019). Generalized `useful' non-symmetric divergence measures and inequalities. *J. Math. Inequal.*, 13(2), 451–466.

[14] Parkash, O. (2000). On fuzzy symmetric divergence. *The Fourth Asian Fuzzy System Symposium*, 2**,** 904-908.

[15] Parkash, O., Sharma, P.K., Kumar, S. (2006). Two new measures of fuzzy divergence and their properties. *SQU Journal for Science*, 11, 69-77.

[16] Renyi, A. (1961). On measures of entropy and information. *Proceedings 4th Berkeley Symposium on Mathematical Statistics and Probability*, 1, 547-561.

[17] Saraswat R.N. and Khatod N. (2020). *"New Fuzzy Divergence Measures, Series, Its Bounds and Applications in Strategic Decision-Making",* In: Kalam A., Niazi K., Soni A., Siddiqui S., Mundra A. (eds) Intelligent Computing Techniques for Smart Energy Systems. Lecture Notes in Electrical Engineering, 607, 641-653, Springer, Singapore.

[18] Torra, V., Narukawa, Y. and Sugeno, M. (2020). On the *f*-divergence for discrete non-additive measures. *Information Sciences*, 512, 50-63.

[19] Zadeh, L.A. (1965). Fuzzy sets. *Information and Control*, 8(3), 338-353.