



Random fixed point theorem use in Fuzzy 3- Metric Space Shows impact of E.A. Property

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Abstract:

The purpose of this research paper we use impact of E.A. property in fuzzy 3-metric space and the condition for two pairs of weakly compatible maps under E.A. like property have unique common fixed point. Our main result E.A. Concept properties, weak compatibility allocation. To show the validity of the main results. The results also applies contraction condition to identify the unique fixed points of Fuzzy 3-Metric Space.

Keywords:Fuzzy 3- metric spaces, common fixed point, weakly compatible maps, common E.A. like property

Introduction:

The contraction mapping principle on complete metric space first seemed in Banach thesis. While dealing with natural world with uncertainty we discover that classical strategies do no longer suffice for this reason a few strategies with a few unique technique with some specific logic are necessitated. In his seminal paper zadeh (1965,[18]) brought the perception of fuzzy sets for huge application. Fuzzy set concept is one of the uncertainty process which assist



tomathematical model well suited to concrete real life situation. The relationship between the fixed point theory and the geometry of fuzzy metric space have been very near and cohesive. Rajput. A et.al (2010,[15]) Common fixed points of Compatible Self Maps in Complete intuitionistic Fuzzy Metric Space. In this paper is to give the new result which is used concept and proved a common fixed point theorem. Rajput. A et.al (2011,[12]) proved Common Fixed point theorems for multivalued maps use in cone intuitionistic fuzzy metric spaces. In this paper to introduce the concept of complete cone metric space for a multivalued transformation. Rajput. A et.al (2011,[14]) proved Common fixed points of compatible maps in intuitionistic fuzzy metric space of integral type. In this paper to obtain a new common fixed point theorems in an intuitionistic fuzzy metric space for point wise R- weakly commuting mappings using contractive condition of integral type.

Alaca et al. (2006,[1]) have established intuitionistic fuzzy versions of Banach contraction principle and Edelstein fixed point theorem. Rajput. A et.al (2012,[5]), proved Common Fixed Points End Point Theorems use in Intuitionistic Fuzzy Metric Spaces. Then presented an endpoint result we initiate end point theory for fuzzy contraction maps in intuitionistic fuzzy metric spaces. Defined a property (E.A) which generalizes the concept of non-compatible mappings and gave some common fixed point theorems under strict contractive conditions. Tripathi. N et.al (2012,[17]) proved Intuitionistic Fuzzy Metric Space Using Concept of α -Fixed Point. In this paper we introduce the notion of common property EA in intuitionistic fuzzy metric spaces with the help of α -fixed point. Rajput. A et.al (2012,[9]) proved Common alpha- Fixed Points Theorems for Multivalued Mappings in intuitionistic Fuzzy Metric Spaces. Recently, Rajput. A et.al (2012,[6]) proved Non Compatible Mappings in intuitionistic fuzzy metric space. Afterward (EAs) property for two pairs of multivalued mappings are introduced and the common fixed point existence theorems and generalized the concept of non compatibility by defining E.A property for self mapping. It contained the class of non compatible mappings in metric spaces. Subsequently a number of fixed point results were proved for contraction mappings satisfying the E.A. property. In this paper we prove common fixed point theorem using common E.A. like property with integral type inequality in fuzzy 3-metric space.



2. PRELIMINARIES

Definition 2.1: A triangle norm $*$ is a binary operation on the unit interval $[0, 1]$ such that for all $a, b, c, d \in [0, 1]$. The following conditions are satisfied

- (i) $a * 1 = a$,
- (ii) $a * b = b * a$
- (iii) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$
- (iv) $a * (b * c) = (a * b) * c$

Definition 2.2

A binary operation $*$: $[0, 1] \times [0, 1] \times [0, 1] \rightarrow [0, 1]$ is a continuous t -norm if $([0, 1], *)$ is an abelian topological monoid with unit 1 such that $a_1 * b_1 * c_1 \leq a_2 * b_2 * c_2$ whenever $a_1 \leq a_2, b_1 \leq b_2, c_1 \leq c_2$ for all a_1, a_2, b_1, b_2 and $c_1, c_2 \in [0, 1]$.

Example of a t -norm are $a * b = ab$ and $a * b = \min \{ a, b \}$

Definition 2.3

A 3-tuple $(X, M, *)$ is said to be fuzzy 2- metric space if X is an arbitrary set, $*$ is a continuous t -norm and M is fuzzy sets in $X^3 \times [0, \infty)$ satisfying the following conditions, for all $x, y, z, u \in X$ and $t_1, t_2, t_3 > 0$.

- (i) $M(x, y, z, 0) = 0$,
- (ii) $M(x, y, z, t) = 1$ for all $t > 0$ if and only if at least two or three points are equal,
- (iii) $M(x, y, z, t) = M(x, z, y, t) = M(y, z, x, t)$
- (iv) $M(x, y, z, t_1 + t_2 + t_3) \geq M(x, y, u, t_1) * M(x, u, z, t_2) * M(u, y, z, t_3)$

(This correspond to tetrahedron inequality in 2- metric space)

The function value $M(x, y, z, t)$ may be interpreted as the probability that the area of triangle is less than t .

- (v) $M(x, y, z, \cdot) : [0, \infty) \rightarrow [0, 1]$ is left continuous.



Definition 2.4

Let $(X, M, *)$ is called a fuzzy 2 metric space:

- (i) A sequence $\{x_n\}$ in fuzzy 2 metric space X is said to be convergent to a point $x \in X$ if $\lim_{n \rightarrow \infty} M(x_n, x, a, t) = 1$, for all a in X and $t > 0$.
- (ii) A Sequence $\{x_n\}$ in fuzzy 2 metric space X is called a Cauchy sequence if $\lim_{n \rightarrow \infty} M(x_{n+p}, x_n, a, t) = 1$, for all a in X and $t > 0, p > 0$.
- (iii) A fuzzy 2 metric space is said to be complete every Cauchy sequence is convergent.

Definition 2.5

A function M is continuous in fuzzy 2 metric space iff whenever $x_n \rightarrow x, y_n \rightarrow y$ then

$$\lim_{n \rightarrow \infty} M(x_n, y_n, a, t) = M(x, y, a, t), \text{ for all } a \in X \text{ and } t > 0.$$

Definition 2.6

Two mappings A and S on a fuzzy 2 metric space X are said to weakly commuting if

$$M(ASx, SAx, a, t) \geq M(Ax, Sx, a, t), \forall x, a \in X \text{ and } t > 0.$$

Definition 2.7 A binary operation $*$: $[0,1]^4 \rightarrow [0,1]$ is called a continuous t-norm if $([0,1], *)$ is an abelian topological monoid with unit 1 such that $a_1 * b_1 * c_1 * d_1 \leq a_2 * b_2 * c_2 * d_2$ whenever $a_1 \leq a_2, b_1 \leq b_2, c_1 \leq c_2, d_1 \leq d_2$ for all $a_1, b_1, c_1, d_1, a_2, b_2, c_2, d_2$ are in $[0,1]$.

Definition 2.8 The 3-tuple $(X, M, *)$ is called a fuzzy 3- metric space if X is an arbitrary set, $*$ is a continuous t-norm and M is a fuzzy set in $X^4 \times [0, \infty]$ satisfying the following conditions for all $x, y, z, u, w \in X$ and $t_1, t_2, t_3, t_4 > 0$.



FM'-1 $M(x, y, z, w, 0) = 0,$

FM'-2 $M(x, y, z, w, t) = 1 \forall t > 0$ iff $x = y,$

(only when three simplex $\{x, y, z, w\}$ degenerate

FM'-3 $M(x, y, z, w, t) = M(x, w, z, y, t) = M(y, z, w, x, t) = M(z, w, x, y, t) =$

...

(Symmetric about three variables)

FM'-4 $M(x, y, z, w, t_1 + t_2 + t_3 + t_4) \geq M(x, y, z, u, t_1) * M(x, y, u, w, t_2)$

$$* M(x, u, z, w, t_3) * M(u, y, z, w, t_4)$$

FM'-5 $M(x, y, z, w, .): [0,1] \rightarrow [0,1]$ is left continuous.

Definition 2.9 Let $(X, M, *)$ is called a fuzzy 3 metric space:

- i. A sequence $\{x_n\}$ in fuzzy 3 metric space X is said to be convergent to a point $x \in X$ if $\lim_{n \rightarrow \infty} M(x_n, x, a, b, t) = 1$, for all a, b in X and $t > 0$.
- ii. A Sequence $\{x_n\}$ in fuzzy 3 metric space X is called a Cauchy sequence if $\lim_{n \rightarrow \infty} M(x_{n+p}, x_n, a, b, t) = 1$, for all a, b in X and $t > 0, p > 0$.
- iii. A fuzzy 3 metric space is said to be complete every Cauchy sequence is convergent.

Definition 2.10 A function M is continuous in fuzzy 3 metric space iff whenever $x_n \rightarrow x, y_n \rightarrow y$ then

$$\lim_{n \rightarrow \infty} M(x_n, y_n, a, b, t) = M(x, y, a, b, t), \text{ for all } a, b \in X \text{ and } t > 0.$$



Definition 2.11 Two mappings A and S on a fuzzy 3 metric space X are said to weakly commuting if

$$M(ASx, SAx, a, b, t) \geq M(Ax, Sx, a, b, t), \forall x, a, b \in X \text{ and } t > 0.$$

Definition 2.12

Let f and g be self mapping from a fuzzy 2-metric space $(X, M, *)$ into itself. A pair of map $\{ f, g \}$ said to be compatible if $\lim_{n \rightarrow \infty} M(fgx_n, gfx_n, a, t) = 1$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n \rightarrow \infty} fx_n = \lim_{n \rightarrow \infty} gx_n = u$ for some u in X and for all $t > 0$.

Definition 2.13

A pair of self mapping $\{ f, g \}$ of a fuzzy 2-metric space $(X, M, *)$ is said to be weakly compatible if they commute at the coincidence points, i.e. $fu = gu$ for some $u \in X$ then $fgu = gfu$.

It is to see that two compatible maps are weakly compatible but converse is not true.

Definition 2.14 Let f and g be two self-maps of a fuzzy metric space $(X, M, *)$. Then they are said to satisfy the E. A. property, if there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n \rightarrow \infty} fx_n = \lim_{n \rightarrow \infty} gx_n = t \text{ for some } t \in X$$

Now in Similar mode we state E.A. property in fuzzy 2-metric spaces as follows

Definition 2.15

A pair of selfmapping $\{ f, g \}$ of a fuzzy 2-metric space $(X, M, *)$ is said to be E.A. property, if there exists a sequence $\{x_n\}$ in X such that



$\lim_{n \rightarrow \infty} M(fx_n, gx_n, a, t) = 1$ for some $t \in X$.

Definition 2.16

Let $A, B, S, T : X \rightarrow X$ where X is a fuzzy 2-metric space, then the pair $\{A, S\}$ and $\{B, T\}$ said to satisfy common E. A. like property if there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n = \lim_{n \rightarrow \infty} By_n = \lim_{n \rightarrow \infty} Ty_n = z$$

where $z \in S(X) \cap T(X)$ or $z \in A(X) \cap B(X)$

Definition 2.17

Let (X, d) be a compatible metric space, $\alpha \in [0, 1]$, $f: X \rightarrow X$ a mapping such that for each $x, y \in X$

$\int_0^{d(fx, fy)} \varphi(t) dt \leq \alpha \int_0^{d(x, y)} \varphi(t) dt$ where $\varphi: R^+ \rightarrow R$ is lebesgue integral mapping which is summable,

$$\varepsilon > 0, \int_0^\varepsilon \varphi(t) dt > 0$$

Nonnegative and such that for each. Then f has unique common fixed $z \in X$ such that for each $x \in X, \lim_{n \rightarrow \infty} f^n x = z$

Rhodes [30], extended this result by replacing the above condition by the following

$$\int_0^{d(fx, fy)} \varphi(t) dt \leq \alpha \int_0^{\max\{d(x, y), d(x, fx), d(y, fy), \frac{1}{2}[d(x, fy) + d(x, fx)]\}} \varphi(t) dt$$



2.18 Lemma: Let $(X, \mathcal{M}, *)$ be a fuzzy 2-metric space. If there exist $k \in (0, 1)$ such that for all $\mathcal{M}(x, y, z, kt) \geq \mathcal{M}(x, y, z, t)$ for all $x, y, z \in X$ with $z \neq x, z \neq y$ and $t > 0$, then $x = y$.

3. MAIN RESULT

Theorem: Let A, B, S and T be self-mappings of a fuzzy 3-metric space $(X, M, *)$ satisfying the following:

(i) For any x, y, a, b in X , and for all $t > 0$ there exists $k \in (0, 1)$ such that

$$M(Ax, By, a, b, kt) \geq \varphi [M(Sx, Ty, a, b, t), M(Ax, Sx, a, b, t), M(By, Ty, a, b, t), M(Sx, By, a, b, t), M(Ax, Ty, a, b, t)]$$

(ii) Pair (A, S) and (B, T) satisfy common E. A. like property.

(iii) Pair (A, S) and (B, T) are weakly compatible. Then A, B, S, T have a unique common fixed point.

Proof: Since (A, S) and (B, T) satisfy E.A. like property therefore there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n = \lim_{n \rightarrow \infty} By_n = \lim_{n \rightarrow \infty} Ty_n = z_1$$

Where $z_1 \in A(X) \cap B(X)$ or $z_1 \in S(X) \cap T(X)$.

Suppose $z_1 \in S(X) \cap Q(X)$, now we have $\lim_{n \rightarrow \infty} Ax_n = z_1 \in S(X)$ then $z_1 = Su$ for some $u \in X$

Since (R, P) and (S, Q) satisfy common E. A. like property therefore there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that

Now we claim that $Au = Su$, put $x = u$ & $y = y_n$ from (1), we have

$$M(Au, By_n, a, b, kt) \geq \varphi [M(Su, Ty_n, a, b, t), M(Au, Su, a, b, t), M(By_n, Ty_n, a, b, t), M(Su, By_n, a, b, t), M(Au, Ty_n, a, b, t)]$$



Taking limit $n \rightarrow \infty$, we have

$$M(Au, z_1, a, b, kt) \geq \varphi [M(z_1, z_1, a, b, t), M(Au, z_1, a, b, t), M(z_1, z_1, a, b, t), M(z_1, z_1, a, b, t), M(Au, z_1, a, b, t)]$$

$$M(Au, z_1, a, b, kt) \geq \varphi [1, M(Au, z_1, a, b, t), 1, M(Au, z_1, a, b, t)]$$

Using lemma 2.18 implies that $Au = z_1 = Su$

Since the pair (A, S) is weakly compatible, so $Az_1 = ASu = SAu = Sz_1$

Again $\lim_{n \rightarrow \infty} By_n = z_1 \in T(X)$ then $z_1 = Tv$ for some $v \in X$

Now we claim that $Bv = z_1$, put $x = x_n$ & $y = v$ then from (1), we have

$$M(Ax_n, Bv, a, b, kt) \geq \varphi [M(Sx_n, Tv, a, b, t), M(Ax_n, Sx_n, a, b, t), M(Bv, Tv, a, b, t), (Sx_n, Bv, a, b, t),$$

$$M(Ax_n, Tv, a, b, t)]$$

Taking limit $n \rightarrow \infty$, we have

$$M(z_1, Bv, a, b, kt) \geq \varphi [M(z_1, z_1, a, b, t), M(z_1, z_1, a, b, t), M(Bv, z_1, a, b, t), M(z_1, Bv, a, b, t), M(z_1, z_1, a, b, t)]$$

$$M(z_1, Bv, a, b, kt) \geq \varphi [1, 1, M(z_1, Bv, a, b, t), M(z_1, Bv, a, b, t), 1]$$

Using lemma 2.18 implies that $Bv = z_1 = Tu$

Since the pair (B,T) is weakly compatible, so $Tz_1 = TBv = BTv = Bz_1$

Now we show that $Az_1 = z_1$, put $x = z_1$ & $y = y_n$ then from (1), we have

$$M(Az_1, By_n, a, b, kt) \geq \varphi [M(Sz_1, Ty_n, a, b, t), M(Az_1, Sz_1, a, b, t), M(By_n, Ty_n, a, b, t), M(Sz_1, By_n, a, b, t), M(Az_1, Ty_n, a, b, t)]$$

Taking limit $n \rightarrow \infty$, we have

$$M(Az_1, z_1, a, b, kt) \geq \varphi [M(Az_1, z_1, a, b, t), M(Az_1, z_1, a, b, t), M(z_1, z_1, a, b, t),$$

$$M(Az_1, z_1, a, b, t), M(Az_1, z_1, a, b, t)]$$

$$M(Az_1, z_1, a, b, kt) \geq \varphi [M(Az_1, z_1, a, b, t), M(Az_1, z_1, a, b, t), 1, M(Az_1, z_1, a, b, t), M(Az_1, z_1, a, b, t)]$$

Using lemma 2.9 implies that $Az_1 = z_1$



Now we show that $Az_1 = z_1$, put $x = x_n$ & $y = z_1$ from (1), we have

$$M(Ax_n, Bz_1, a, b, kt) \geq \varphi [M(Sx_n, Tz_1, a, b, t), M(Ax_n, Sx_n, a, b, t), M(Bz_1, Tz_1, a, b, t), \\ M(Sz_1, Bz_1, a, b, t), M(Ax_n, Tz_1, a, b, t)]$$

Taking limit $n \rightarrow \infty$, we have

$$M(z_1, Bz_1, a, b, kt) \geq \varphi [M(z_1, z_1, a, b, t), M(z_1, z_1, a, b, t), M(Bz_1, z_1, a, b, t), \\ M(z_1, Bz_1, a, b, t), M(z_1, z_1, a, b, t)]$$

$$M(z_1, Bz_1, a, b, kt) \geq \varphi [1, 1, M(Bz_1, z_1, a, b, t), M(z_1, Bz_1, a, b, t), 1]$$

Using lemma 2.18 implies that $Bz_1 = z_1$

$$Az_1 = Sz_1 = Bz_1 = Tz_1 = z_1$$

Thus z_1 is common fixed point of A, B, S and T.

4 Conclusion:

The purpose of this research paper strengthens the results and to emphasize the role of property E.A. in the existence of common fixed points and prove the main result for a pair of weakly compatible mappings along with E.A. Property

There are four improvements in this paper

1. To relax the continuity requirements of maps completely.
2. To minimize the commutativity requirement of the maps to the point coincidence.
3. To weaken the completeness requirement of the space.
4. Property of E.A. buys containment of ranges without any continuity requirements to the point coincidence.

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