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RANKING OF HEPTAGONAL FUZZY NUMBERS USING INCENTRE OF CENTROIDS

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ABSTRACT

The fuzzy set theory has been applied in almost every business enterprise as well as day to day activity. Ranking of fuzzy numbers plays an important role in decision making process. In this paper, a ranking method for heptagon fuzzy numbers based on area is proposed. The study involves computation of centroid of centroids and incentre of centroids for ranking of heptagonal fuzzy numbers. Proposed method is illustrated with the help of examples.

***Keywords*: Heptagonal fuzzy numbers, centroid of centroids, Incentre, Ranking Function.**

I. INTRODUCTION

The concept of fuzzy set theory deals with imprecision, vagueness in real life situations. It was firstly proposed by Zadeh [1]. Bellman and Zadeh [2] elaborated on the concept of decision making in the fuzzy environment. Later on, fuzzy methodologies have been successfully applied in a wide range of real world situations. Jain [3] was the first to propose method of ranking fuzzy numbers for decision making in fuzzy situations. Yager [4] used the concept of centroids in the ranking of fuzzy numbers. Chu and Tsao [5] computed the area between the original point and the centroid point for ranking of fuzzy numbers. Babu et al. [6] described the method for ranking various types of fuzzy numbers and crisp numbers based on area, mode, spreads and weights of generalized trapezoidal fuzzy numbers. They also apply mode and spread in those cases where the discrimination is not possible. Dhanalaxmi and Kennedy [7] proposed some ranking methods for octagonal fuzzy numbers. Rajarajeshwari and Sudha [8] explained ordering generalized hexagonal fuzzy numbers by using rank, mode, divergence and spread. Thorani et al. [9] illustrated ordering of generalized trapezoidal fuzzy numbers by using orthocenter of centroids. Kumar et al. [10] introduced some additional assumptions for the equality of generalised triangular fuzzy numbers. Monsavi and Regvani [11] proposed a method for ranking of generalised triangular fuzzy numbers which are based on rank, mode, divergence, and spread. Gani and Assarudeen[12] modified the method of subtraction and division of triangular fuzzy numbers. Kumar et al [13] proposed a approach for ranking of generalised trapezoidal fuzzy number based on rank, mode, divergence and spread.

Abbasbandy and Hajjari [14] introduced a new approach for ranking based on the left and the right spreads at some α-levels of trapezoidal fuzzy numbers. Roseline and Amirtharaj [15] proposed a improved method of ranking of generalised trapezoidal fuzzy numbers based on rank, mode,divergence and spread. Rajarajeswari

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and Sudha [16] proposed a new method for ranking of fuzzy numbers by using incentre of centroids. Rajarajeswari et al [17] introduced new operations for addition, subtraction and multiplication of hexagonal fuzzy numbers on the basis of α-cut. Namarta and Neha[18] proposed ranking of hendecagonal fuzzy numbers by using centroid of centroids. Jatinder and Neha [19] proposed the ranking method for ordering dodecagonal fuzzy numbers based on rank, mode, divergence and spread.

Jatinder and Neha [20] described a Ranking method for ordering dodecagonal fuzzy numbers by using incentre of centroids.

There are various papers on methods of ranking of triangular fuzzy numbers, trapezoidal fuzzy numbers, hexagonal fuzzy numbers, octagonal fuzzy numbers etc.

The present paper describes the method of ranking heptagonal fuzzy numbers using centroid of centroids and incentre of centroids. In a heptagon fuzzy number, firstly the heptagon is split into two trapezoidal and one rhombus and compute the centroid of these plane figures. Secondly, it computes the centroid of these centroids and the centroid is followed by calculation of incentre. In this paper, the method of ranking fuzzy numbers with an area between the incentre and the original point is also proposed and when discrimation is not possible mode , divergence and spread is also used.

The paper is organized as follows: In Section 2, some basic definitions of fuzzy set and heptagon fuzzy numbers are given. Section 3 presents proposed ranking method that deals with the computational procedure of centroid of centroids and incentre of centroid . In Section 4, some important results are discussed that are useful for proposed approach. In Section 5, proposed method is illustrated with the help of numerical examples. Finally conclusion is presented in Section 6.

II. PRELIMINARIES

In this section, definitions of fuzzy set, fuzzy numbers and hendecagonal fuzzy numbers are presented.

Definition 2.1 Let X = {x} denote a collection of objects denoted generically by x. Then a fuzzy set $\widetilde{A}$ in X is a set of ordered pairs $\widetilde{A}$ = {(x, $\mu_{\tilde{A}}$ (x)); x ∈ X} where $\mu_{\tilde{A}}$(x) is termed as the grade of membership of x in A and $\mu_{\tilde{A}}$: X → M is a function from X to a space M which is called membership space. When M contains only two points, 0 and 1, A is non fuzzy and its membership function becomes identical with the characteristic function of a non fuzzy set.

Definition 2.2 A Fuzzy set $\widetilde{A}$ of universe set X is normal if and only if $Sup_{x\epsilon X}\ \ \mu_{\tilde{A}}$ (x) = 1.

Definition 2.3 A fuzzy set $\widetilde{A}$ in universal set X is called convex iff

$\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \ \geq \min[\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)]$ for all $x_1, x_2 \in$ X and $\lambda \in [0,1]$.

Definition 2.4 A fuzzy set $\widetilde{A}$ of universal set is a fuzzy number iff it is normal and convex.

Definition 2.5 A fuzzy number

$\widetilde{A}$ = (m, n, α, β)$_{LR}$ is said to be an LR flat fuzzy number if its membership function is given by:

$$\mu_{\tilde{A}}(x) = \begin{Bmatrix} L\left(\frac{m-x}{\alpha}\right), & x \leq m, \alpha > 0 \\ R\left(\frac{x-n}{\beta}\right), & x \geq n, \beta > 0 \\ 1, & m \leq x \leq n \end{Bmatrix}$$

L and R are called reference functions, which are continuous, non-increasing functions that defining the left and right shapes of $\mu_{\tilde{A}}$ (x) respectively and L(0)=R (0) =1.

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Definition 2.6. Heptagonal Fuzzy Numbers:

A generalised fuzzy number

$\widetilde{A_H}$ =$(a_1, a_2, a_3, a_4, a_5, a_6, a_7; w)$ is said to be heptagonal fuzzy number if its membership function $\mu_{\widetilde{A_H}}$(x) is given below:

$$\mu_{\widetilde{A_H}}(x) = \begin{cases} 0 & x \le a_1 \\ \frac{w}{2}\left(\frac{x-a_1}{a_2-a_1}\right) & a_1 \le x \le a_2 \\ \left(\frac{w}{2}\right) & a_2 \le x \le a_3 \\ \frac{w}{2}+\frac{w}{2}\left(\frac{x-a_3}{a_3-a_2}\right) & a_3 \le x \le a_4 \\ w & x = a_4 \\ \frac{w}{2}+\frac{w}{2}\left(\frac{a_5-x}{a_5-a_4}\right) & a_4 \le x \le a_5 \\ \left(\frac{w}{2}\right) & a_5 \le x \le a_6 \\ \frac{w}{2}\left(\frac{a_7-x}{a_7-a_6}\right) & a_6 \le x \le a_7 \\ 0 & x \ge a_7 \end{cases}$$

III. PROPOSSED RANKING METHOD

The Centroid of Heptagonal fuzzy numbers is considered to be the balancing point of the heptagon. Firstly, divide the heptagon into two trapezoidal and one rhombus AHIC, DKLG and IJKD respectively. and then find the centroid of these plane figures. Let the centroid of these plane figures be G_1, G_2 *and* G_3 respectively. The Centroid of centroids G_1, G_2 *and* G_3 is taken as the point of reference to define the ranking of generalized heptagonal fuzzy numbers. The incentre of centroids G_1, G_2 *and* G_3 would be a better point than the centroid point of the Heptagon. Consider the generalized heptagonal fuzzy number $\hat{A}_H = (a_1, a_2, a_3, a_4, a_5, a_6, a_7;\ w)$

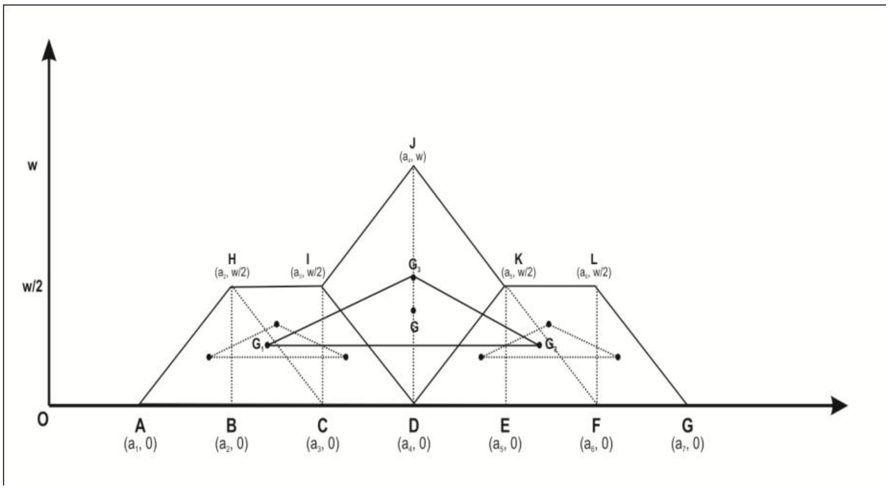

Fig.1

The centroid of these triangles are

$G_1 = (\frac{2a_1+7a_2+7a_3+2a_4}{18}, \frac{7w}{36})$;

$G_2 = (\frac{2a_4+7a_5+7a_6+2a_7}{18}, \frac{7w}{36})$;

$G_3 = (a_4, \frac{w}{2})$

$As\ \ G_1, G_2\ and\ G_3$ are non collinear and they form a triangle. Therefore Centroid $G_{\widetilde{A_H}}(\overline{x_0}\ ,\ \overline{y_0})$ of a triangle whose vertices are $G_1, G_2\ and\ G_3$ of generalized heptagonal fuzzy numbers is

$G_{\widetilde{A_H}}(\overline{x_0}\ ,\ \overline{y_0})= (\frac{2a_1+7a_2+7a_3+22a_4+7a_5+7a_6+2a_7}{54}\ ,\ \frac{11w}{54})$

The Ranking function of the generalized heptagonal fuzzy numbers is defined as :

$R_{\widetilde{A_H}}(\bar{x}_0, \bar{y}_0) = \bar{x}_0 . \bar{y}_0 = \ (\frac{2a_1+7a_2+7a_3+22a_4+7a_5+7a_6+2a_7}{54})(\frac{11w}{54})$

Now we define the Incentre $I_{\widetilde{A_H}}(\overline{x_0}\ ,\ \overline{y_0})$ of a triangle with vertices G_1, G_2 and G_3 of a generalized heptagonal fuzzy number is given by :

$I_{\widetilde{A_H}}(\overline{x_0}\ ,\ \overline{y_0}) = (a_{\widetilde{A_H}}\left(\frac{2a_1+7a_2+7a_3+2a_4}{18}\right) +\ b_{\widetilde{A_H}}\left(\frac{2a_4+7a_5+7a_6+2a_7}{18}\right) + c_{\widetilde{A_H}}(\ a_4),\ a_{\widetilde{A_H}}\left(\frac{7w}{36}\right) +\ b_{\widetilde{A_H}}\left(\frac{7w}{36}\right) + c_{\widetilde{A_H}}\left(\frac{w}{2}\right))$

Where $a_{\widetilde{A_H}} = \frac{\sqrt{(32a_4-14a_5-14a_6-4a_7)^2+(11w)^2}}{36}$

$$b_{\widetilde{A_H}} = \frac{\sqrt{(32a_4 - 4a_1 - 14a_2 - 14a_3)^2 + (11w)^2}}{36}$$

$$c_{\widetilde{A_H}} = \frac{-2a_1 - 7a_2 - 7a_3 + 7a_5 + 7a_6 + 2a_7}{18}$$

The Ranking function of the generalized heptagonal fuzzy number is defined as:

$R\ (\widetilde{A_H}) = \sqrt{\overline{x_0}^2 + \overline{y_0}^2}$

This is area between the incentre of centroids and the original point.

The Mode of generalized heptagonal fuzzy number

$\widetilde{H_D} =(a_1, a_2, a_3, a_4, a_5, a_6, a_7; w)$ is defined as:

Mode (M) = $\frac{1}{2}\int_0^w\ a_4\ dx\ = \frac{m a_4}{2}$

The Divergence of generalized heptagonal fuzzy number

$\widetilde{H_D} =(a_1, a_2, a_3, a_4, a_5, a_6, a_7; w)_{LR}$ is defined as:

Divergence (D) = $w(a_7 - a_1)$

The Left Spread of generalized heptagonal fuzzy number

$\widetilde{H_D} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7; w)$ is defined as:

Left Spread (LS) = $w(a_4 - a_1)$

The Right Spread of generalized heptagonal fuzzy number

$\widetilde{H_D} =(a_1, a_2, a_3, a_4, a_5, a_6, a_7; w)$ is defined as:

Right Spread (RS) = $w(a_7 - a_4)$

IV. PROPOSED APPROACH FORRANKING OF GENERALISED HEPTAGON FUZZY NUMBERS

Using the above definitions proposed in section 3. the ranking procedure for two generalized heptagonal fuzzy numbers is as follows:

Let $\tilde{A} =(a_1, a_2, a_3, a_4, a_5, a_6, a_7;\ w_1\ )$ and

$\tilde{B} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7;\ w_2)$ be two generalized heptagon fuzzy numbers then

Step 1. Find $R(\tilde{A})$ *and* $R(\tilde{B})$

1) If $R(\tilde{A}) > R(\tilde{B})$ then $\tilde{A} > \tilde{B}$

2) If $R(\tilde{A}) < R(\tilde{B})$ then $\tilde{A} < \tilde{B}$

3) If $R(\tilde{A}) = R(\tilde{B})$, comparison is not possible then go to step 2.

Step 2. Find $M(\tilde{A})$ *and* $M(\tilde{B})$

1) If $M(\tilde{A}) > M(\tilde{B})$ then $\tilde{A} > \tilde{B}$

2) If $M(\tilde{A}) < M(\tilde{B})$ then $\tilde{A} < \tilde{B}$

3) If $M(\tilde{A}) = M(\tilde{B})$, comparison is not possible then go to step 3.

Step 3. Find $D(\tilde{A})$ *and* $D(\tilde{B})$

1) If $D(\tilde{A}) > D(\tilde{B})$ then $\tilde{A} > \tilde{B}$

2) If $D(\tilde{A}) < D(\tilde{B})$ then $\tilde{A} < \tilde{B}$

3) If $D(\tilde{A}) = D(\tilde{B})$, comparison is not possible then go to step 4.

Step 4. Find $LS(\tilde{A})$ *and* $LS(\tilde{B})$

If $LS(\tilde{A}) > LS(\tilde{B})$ then $\tilde{A} > \tilde{B}$

1) If $LS(\tilde{A}) < LS(\tilde{B})$ then $\tilde{A} < \tilde{B}$

2) If $LS(\tilde{A}) = LS(\tilde{B})$ then go to step 5.

Step 5. Find w_1 and w_2

a) If $w_1 > w_2$ then $\tilde{A} > \tilde{B}$

b) If $w_1 < w_2$ then $\tilde{A} < \tilde{B}$

c) If $w_1 = w_2$ then $\widetilde{A} \sim \tilde{B}$

V. NUMERICAL PROBLEMS

5.1 Example

Let $\widetilde{A_H}$ = (0.2, 0.4, 0.6, 0.8, 1, 1.2, 1.4 ; 1)

and $\widetilde{B_H}$ = (0.2, 0.5, 0.6, 0.9, 1.1, 1.2, 1.4; 1) be two generalized heptagonal fuzzy numbers

Then $G_{\widetilde{A_H}}(\overline{x_0}, \overline{y_0})$ = (0.670, 0.2037)

$G_{\widetilde{B_H}}(\overline{x_0}, \overline{y_0})$ = (0.867, 0.2037)

$I_{\widetilde{A_H}}(\overline{x_0}, \overline{y_0})$ = (0.766, 0.329)

Where $a_{\widehat{A_H}} = 0.428, b_{\widehat{A_H}} = 0.428\ c_{\widehat{A_H}} = 0.68$

$I_{\widetilde{B_H}}(\overline{x_0}, \overline{y_0}) = (0.836, 0.3065)$

Where $a_{\widehat{B_H}} = 0.3947, b_{\widehat{B_H}} = 0.362, c_{\widehat{B_H}} = 0.444$

$R(\widetilde{A_H})$ = 0.8338 and $R(\widetilde{B_H})$ = 0.8999

Thus $R(\widetilde{A_H}) < R(\widetilde{B_H})$ then $\widetilde{A_H} < \widetilde{B_H}$

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5.2 Example

Let $\widetilde{A_H}$= (0.2, 0.4, 0.6, 0.9, 1.1, 1.4, 1.7 ; 1)

and $\widetilde{B_H}$ = (0.1, 0.3, 0.8, 0.9, 1.1, 1.3, 1.8 ; 1) be two generalized heptagonal fuzzy numbers

Then $G_{\widetilde{A_H}}(\overline{x_0}, \overline{y_0})$ = (0.8907, 0.2037)

$G_{\widetilde{B_H}}(\overline{x_0}, \overline{y_0})$ = (0.8907, 0.2037)

$I_{\widetilde{A_H}}(\overline{x_0}, \overline{y_0})$ = (0.896, 0.828)

Where $a_{\widehat{A_H}} = 0.4729, b_{\widehat{A_H}} = 0.4945, c_{\widehat{A_H}} = 0.75$

$I_{\widetilde{B_H}}(\overline{x_0}, \overline{y_0}) = (0.896 , 0.835)$

Where $a_{\widehat{B_H}} = 0.452, b_{\widehat{B_H}} = 0.473, c_{\widehat{B_H}} = 0.69$

Step 1 Find $R(\widetilde{A_H})$ and $R(\widetilde{B_H})$

$R(\widetilde{A_H})$ = 1.220 and $R(\widetilde{B_H})$ = 1.224

Thus $R(\widetilde{A_H}) = R(\widetilde{B_H})$ then go to step 2

Step 2 Find $M(\widetilde{A_H})$ and $M(\widetilde{B_H})$

$M(\widetilde{A_H})$ = 0.45 and $M(\widetilde{B_H}) =$ 0.45

Since $M(\widetilde{A_H}) = M(\widetilde{B_H})$ then go to step 3

Step 3 Find $D(\widetilde{A_H})$ and $D(\widetilde{B_H})$

$D(\widetilde{A_H})$ = 1.5 and $D(\widetilde{B_H}) =$ 1.7

Thus $D(\widetilde{A_H}) < D(\widetilde{B_H}) \Rightarrow \widetilde{A_H} < \widetilde{B_H}$.

5.3 Example

Let $\widetilde{A_H}$= (0.02, 0.03, 0.04, 0.06, 0.08, 0.09, 0.11 ; 0.4)

and $\widetilde{B_H}$ =(0.01, 0.02, 0.05, 0.06, 0.07, 0.10, 0.12 ; 0.4) be two generalized heptagonal fuzzy numbers

Then $G_{\widetilde{A_H}}(\overline{x_0}, \overline{y_0})$ = (0.06037, 0.08148)

$G_{\widetilde{B_H}}(\overline{x_0}, \overline{y_0})$ = (0.06037, 0.08148)

$I_{\widetilde{A_H}}(\overline{x_0}, \overline{y_0})$ = (0.0612, 0.0809)

Where $a_{\widehat{A_H}} = 0.8213, b_{\widehat{A_H}} = 0.880, c_{\widehat{A_H}} = 0.049$

$I_{\widetilde{B_H}}(\overline{x_0}, \overline{y_0}) = (0.06128 , 0.07607)$

Where $a_{\widehat{B_H}} = 0.8202, b_{\widehat{B_H}} = 0.8708, c_{\widehat{B_H}} = 0.051$

Step 1 Find $R(\widetilde{A_H})$ and $R(\widetilde{B_H})$

$R(\widetilde{A_H})$ = 0.1012 and $R(\widetilde{B_H})$ = 0.1012

Thus $R(\widetilde{A_H}) = R(\widetilde{B_H})$ then go to step 2

Step 2 Find $M(\widetilde{A_H})$ and $M(\widetilde{B_H})$

$M(\widetilde{A_H})$ = 0.12 and $M(\widetilde{B_H}) =$ 0.12

Since $M(\widetilde{A_H}) = M(\widetilde{B_H})$ then go to step 3

Step 3 Find $D(\widetilde{A_H})$ and $D(\widetilde{B_H})$

$D(\widetilde{A_H})$ = 0.036 and $D(\widetilde{B_H}) =$ 0.044

Thus $D(\widetilde{A_H}) < D(\widetilde{B_H}) \Rightarrow \widetilde{A_H} < \widetilde{B_H}$.

5.4 Example

Let $\widetilde{A_H}$ = (1.1, 1.4, 1.6, 1.9, 2.1, 2.4, 2.9 ; 0.8)

and $\widetilde{B_H}$ = (1.1, 1.3, 1.7, 1.9, 2.2, 2.3, 2.9 ; 0.8) be two generalized heptagonal fuzzy numbers

Then $G_{\widetilde{A_H}}(\overline{x_0}, \overline{y_0})$ = (1.894, 0.1629)

$G_{\widetilde{B_H}}(\overline{x_0}, \overline{y_0})$ = (1.894, 0.1629)

$I_{\widetilde{A_H}}(\overline{x_0}, \overline{y_0})$ = (1.940, 0.2630)

Where $a_{\widetilde{A_H}} = 0.46168$, $b_{\widetilde{A_H}} = 0.5342$, $c_{\widetilde{A_H}} = 0.783$

$I_{\widetilde{B_H}}(\overline{x_0}, \overline{y_0}) = (1.897, 0.264)$

Where $a_{\widetilde{B_H}} = 0.46168$, $b_{\widetilde{B_H}} = 0.4687$, $c_{\widetilde{B_H}} = 0.783$

Step 1 Find $R(\widetilde{A_H})$ and $R(\widetilde{B_H})$

$R(\widetilde{A_H})$ = 1.957 and $R(\widetilde{B_H})$ = 1.957

Thus $R(\widetilde{A_H}) = R(\widetilde{B_H})$ then go to step 2

Step 2 Find $M(\widetilde{A_H})$ and $M(\widetilde{B_H})$

$M(\widetilde{A_H})$ = 0.76 and $M(\widetilde{B_H}) = 0.76$

Since $M(\widetilde{A_H}) = M(\widetilde{B_H})$ then go to step 3

Step 3 Find $D(\widetilde{A_H})$ and $D(\widetilde{B_H})$

$D(\widetilde{A_H})$ = 1.44 and $D(\widetilde{B_H}) = 1.44$

Thus $D(\widetilde{A_H}) = D(\widetilde{B_H})$ then go to step 4

Step 4 Find $LS(\widetilde{A_H})\ and\ LS(\widetilde{B_H})$

$LS(\widetilde{A_H})$ = 0.64 and $LS(\widetilde{B_H})$ = 0.64

Since $LS(\widetilde{A_H}) = LS(\widetilde{B_H})$ then go to step 5

Step 5 $w_1 = 0.8\ and\ w_2 = 0.8$

Since $w_1 = w_2$ then $\widetilde{A_H} \sim \widetilde{B_H}$.

VI. CONCLUSION

The paper describes ranking method for heptagonal fuzzy numbers based on area. The process of ranking involves computation of centroids of two trapezoidal figures and one rohmbus formed by heptagon numbers, then obtaining centroid of these centroids . Finally, the centroid of centroids and incenter of centroids is used to rank heptagonal fuzzy numbers and illustrated by an example.

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