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AN OVERVIEW ON INTUITIONISTIC FUZZY SIMILARITY MEASURES

Anshu Ohlan

Research Scholar, Department of Mathematics, Deenbandhu Chhotu Ram University of Science and Technology, Murthal, Haryana, (India)

ABSTRACT

Atanassov's intuitionistic Fuzzy Set (IFS) theory is a convenient tool to handle with uncertainty and vagueness. Although several intuitionistic fuzzy similarity measures have been proposed by the researchers in the past decades. In the present paper we review some of the select studies on similarity measures between intuitionistic fuzzy sets.

Keywords: Fuzziness, Intuitionistic Fuzzy Sets (IFSs), Similarity Measure

I. INTRODUCTION

The fuzzy set theory introduced by Zadeh [1] has received a vital attention of academia for its application in various fields such as pattern recognition, image processing, speech recognition, bioinformatics, fuzzy aircraft control, feature selection, decision making, etc. The significant studies [2-15] made efforts to define measures of information in the fuzzy environment and find their applications in a variety of fields. Similarity measures between fuzzy sets, as an important concept in fuzzy mathematics, have gained vital attention for their wide applications in real world [16-23].

The notion of Atanassov's intuitionistic fuzzy sets (IFSs) was first originated by Atanassov [24] which found to be well suited to deal with both fuzziness and lack of knowledge or non-specificity. It is noted that the concept of an IFS is the best alternative approach to define a FS in cases where existing information is not enough for the definition of imprecise concepts by means of a conventional FS. Thus, the concept of Atanassov IFSs is the generalization of the concept of FSs. In 1993, Gau and Buehrer [25] introduced the notion of vague sets. But, Bustince and Burillo [26] presented that the notion of vague sets was equivalent to that of Atanassov IFSs. For determining the similarity between two IFSs similarity measure is an important tool among the most exciting measures in IFSs theory.

The remainder of the paper is organized as follows. Section 2 is devoted to introduce some well-known concepts, and the notation related to fuzzy set theory, intuitionistic fuzzy set theory and definition of similarity measure in intuitionistic fuzzy set theory. In Section 3, we give an overview on recent development in intuitionistic fuzzy similarity measures. The final section concludes the paper.

II. PRELIMINARIES

We begin by reviewing some well-known concepts related to fuzzy set theory and intuitionistic fuzzy set theory.

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Definition 1. *Fuzzy Set (FS)* [1]: A fuzzy set A' defined on a finite universe of discourse $X=(x_1,x_2,...,x_n)$ is given as:

$$A'=\{\langle x,\mu_{A'}(x)\rangle / x\in X\} \tag{1}$$

where $\mu_{A'}:X\to[0,1]$ is the membership function of A'. The membership value $\mu_{A'}(x)$ describes the degree of the belongingness of $x\in X$ in A'. When $\mu_{A'}(x)$ is valued in {0, 1}, it is the characteristic function of a crisp i.e., non-fuzzy set.

Atanassov [24] introduced the concept of intuitionistic fuzzy set (IFS) as the generalization of the concept of fuzzy set.

Definition 2. *Intuitionistic Fuzzy Set (IFS)* [24]: An Atanassov intuitionistic fuzzy set (IFS) A on a finite universe of discourse $X=(x_1,x_2,...,x_n)$ is defined as

$$A=\{\langle x,\mu_A(x),\nu_A(x)\rangle / x\in X\} \tag{2}$$

where $\mu_A:X\to[0,1]$, $\nu_A:X\to[0,1]$ with the condition $0\le\mu_A+\nu_A\le1\ \forall x_i\in X$.

The numbers $\mu_A(x_i),\nu_A(x_i)\in[0,1]$ denote the degree of membership and non-membership of x_i to A, respectively.

For each intuitionistic fuzzy set in X we will call $\pi_A(x_i)=1-\mu_A(x_i)-\nu_A(x_i)$, the intuitionistic index or degree of hesitation of x_i in A. It is obvious that $0\le\pi_A(x_i)\le1$ for each $x_i\in X$. For a fuzzy set A' in X , $\pi_A(x_i)=0$ when $\nu_A(x_i)=1-\mu_A(x_i)$. Thus, FSs are the special cases of IFSs.

Atanassov [24] further defined set operations on intuitionistic fuzzy sets as follows:

Let $A,B\in IFS(X)$ given by

$$A=\{\langle x_i,\mu_A(x_i),\nu_A(x_i)\rangle / x_i\in X\},$$

$$B=\{\langle x_i,\mu_B(x_i),\nu_B(x_i)\rangle / x_i\in X\},$$

(i) $A\subseteq B$ iff $\mu_A(x_i)\le\mu_B(x_i)$ and $\nu_A(x_i)\ge\nu_B(x_i)\ \forall x_i\in X$.

(ii) $A=B$ iff $A\subseteq B$ and $B\subseteq A$.

(iii) $A^c=\{\langle x_i,\nu_A(x_i),\mu_A(x_i)\rangle / x_i\in X\}$.

(iv) $A\cup B=\{\langle x_i,\max(\mu_A(x_i),\mu_B(x_i)),\min(\nu_A(x_i),\nu_B(x_i))\rangle / x_i\in X\}$.

(v) $A\cap B=\{\langle x_i,\min(\mu_A(x_i),\mu_B(x_i)),\max(\nu_A(x_i),\nu_B(x_i))\rangle / x_i\in X\}$.

A similarity measure between two IFSs A and B is assumed by Hung and Yang [35], Tan and Chen [41] and Chen and Chang [39] to satisfy the following properties:

(i) $0\le S(A,B)\le1$

(ii) $S(A,B)=1$ if and only if $A=B$.

(iii) $S(A,B) = S(B,A)$

(iv) If $A \subseteq B \subseteq C$, $A,B,C \in IFSs(X)$

Then $S(A,C) \le S(A,B)$ and $S(A,C) \le S(B,C)$.

III. RECENT DEVELOPMENTS IN INTUITIONISTIC FUZZY SIMILARITY MEASURES

In this section we present an overview of some of the intuitionistic fuzzy similarity measures existing in the literature. Table 1 briefly reviewed the select similarity measures in intuitionistic fuzzy theory introduced by researchers during last three decades.

Table 1 Similarity measures existing in the literature

Sr. No.	**Author(s)**	**Similarity Measures**
1.	Chen [27]	$S_C(A,B) = 1 - \frac{\sum_{i=1}^{n} \lvert (\mu_A(x_i) - \nu_A(x_i)) - (\mu_B(x_i) - \nu_B(x_i)) \rvert}{2n}$
2.	Hong and Kin [28]	$S_H(A,B) = 1 - \frac{\sum_{i=1}^{n} (\lvert \mu_A(x_i) - \mu_B(x_i) \rvert + \lvert (\nu_A(x_i) - \nu_B(x_i)) \rvert}{2n}$
3.	Li and Xu [29]	$S_L(A,B) = 1 - \frac{\sum_{i=1}^{n} \lvert (\mu_A(x_i) - \nu_A(x_i)) - (\mu_B(x_i) - \nu_B(x_i)) \rvert}{4n} + \frac{\sum_{i=1}^{n} (\lvert \mu_A(x_i) - \mu_B(x_i) \rvert + \lvert (\nu_A(x_i) - \nu_B(x_i)) \rvert}{4n}$
4.	Li and Cheng [30]	$S_{DC}(A,B) = 1 - \sqrt[p]{\frac{\sum_{i=1}^{n} \lvert \varphi_A(x_i) - \varphi_B(x_i) \rvert^p}{n}}$
5.	Dengfeng and Chuntian [31]	$S_D(A,B) = 1 - \sqrt[p]{\frac{\sum_{i=1}^{n} \left\lvert \left(\frac{\mu_A(x_i) + 1 - \nu_A(x_i)}{2} \right) - \left(\frac{\mu_B(x_i) + 1 - \nu_B(x_i)}{2} \right) \right\rvert^p}{n}}$

6.	Mitchell [32]	$S_M(A,B)=\frac{1}{2}\left(1-\sqrt[p]{\frac{\sum_{i=1}^{n}\lvert\mu_A(x_i)-\mu_B(x_i)\rvert^p}{n}}+1-\sqrt[p]{\frac{\sum_{i=1}^{n}\lvert\nu_A(x_i)-\nu_B(x_i)\rvert^p}{n}}\right)$
7.	Li et al.[33]	$S_O(A,B)=1-\sqrt[p]{\frac{\sum_{i=1}^{n}\left(\left(\mu_A(x_i)-\mu_B(x_i)\right)^2+\left(\nu_A(x_i)-\nu_B(x_i)\right)^2\right)}{2n}}$
8.	Liang and Shi [34]	$S_e^p(A,B)=1-\sqrt[p]{\frac{\sum_{i=1}^{n}\left(\phi_\mu(x_i)+\phi_\nu(x_i)\right)^p}{n}}$
9.	Liang and Shi [34]	$S_s^p(A,B)=1-\sqrt[p]{\frac{\sum_{i=1}^{n}\lvert(\psi_{s1}(x_i)+\psi_{s2}(x_i))\rvert^p}{n}}$
10.	Liang and Shi [34]	$S_h^p(A,B)=1-\sqrt[p]{\frac{\sum_{i=1}^{n}\left(\eta_1(x_i)+\eta_2(x_i)+\eta_3(x_i)\right)^p}{3n}}$
11.	Hung and Yang [35]	$S_{HY}^1(A,B)=1-\frac{\sum_{i=1}^{n}\max\left(\lvert\mu_A(x_i)-\mu_B(x_i)\rvert,\lvert\nu_A(x_i)-\nu_B(x_i)\rvert\right)}{n}$
12.	Ye [36]	$C_{IFS}(A,B)=\frac{1}{n}\sum_{i=1}^{n}\frac{\mu_A(x_i)\mu_B(x_i)+\nu_A(x_i)\nu_B(x_i)}{\sqrt{(\mu_A(x_i))^2+(\nu_A(x_i))^2}\sqrt{(\mu_B(x_i))^2+(\nu_B(x_i))^2}}$
13.	Boran and Akay [37]	$S_t^p(A,B)=1-\left(\sum_{i=1}^{n}\frac{1}{2n(1+p)}\left\{\begin{array}{l}\lvert t(\mu_A(x_i)-\mu_B(x_i))-(\nu_A(x_i)-\nu_B(x_i))\rvert^p\\+\lvert t(\nu_A(x_i)-\nu_B(x_i))-(\mu_A(x_i)-\mu_B(x_i))\rvert^p\end{array}\right\}\right)^{1/p}$
14.	Song et al. [38]	$S_Y(A,B)=\frac{1}{2n}\sum_{i=1}^{n}\left(\begin{array}{r}\sqrt{\mu_A(x_i)\mu_B(x_i)}+2\sqrt{\nu_A(x_i)\nu_B(x_i)}+\sqrt{\pi_A(x_i)\pi_B(x_i)}\\+\sqrt{(1-\nu_A(x_i))(1-\nu_B(x_i))}\end{array}\right)$
15.	Chen and Chang [39]	$S_{CC}(A,B)=\sum_{i=1}^{n}\left(w_i\times s(A_{x_i},B_{x_i})\right),w_i\in[0,1],\sum_{i=1}^{n}w_i=1$

		$$s(A_{x_i},B_{x_i})=1-\left(\left\|\mu_A(x_i)-\mu_B(x_i)\right\|\times(1-\frac{\pi_A(x_i)+\pi_B(x_i)}{2})\right)$$ where $$-\left(\left(\frac{\pi_A(x_i)+\pi_B(x_i)}{2}\right)\times\int_0^1\left\|\mu_{A_{x_i}(z)}-\mu_{B_{x_i}(z)}\right\|dz\right)$$
16.	Nguyen [40]	$$S_F(A,B)=\begin{cases}1-\left\|K_F(A)-K_F(B)\right\| \; for \; \tilde{K}_F(A).\tilde{K}_F(B)\geq 0\\ \left\|K_F(A)-K_F(B)\right\|-1 \; for \; \tilde{K}_F(A).\tilde{K}_F(B)<0\end{cases}$$ where $$K_F(A)=\frac{1}{n\sqrt{2}}\sum_{i=1}^{n}\sqrt{(\mu_A(x_i))^2+(\nu_A(x_i))^2+(1-\pi_A(x_i))^2}$$ $$and \;\; K_F(B)=\frac{1}{n\sqrt{2}}\sum_{i=1}^{n}\sqrt{(\mu_B(x_i))^2+(\nu_B(x_i))^2+(1-\pi_B(x_i))^2}$$

It is clear from Table 1 that Chen [30] first defined some similarity measures between vague sets. Later on Hong and Kim [31] pointed out some unreasonable cases of Chen's measure and introduced one modified measure of similarity. Thereafter, Li and Chang [33] proposed new similarity measures and also provide their application with pattern recognition problems. In the continuation of the process of defining the similarity measures between intuitionistic fuzzy sets several measures [34-43] have been defined by the researchers in the last three decades with some drawbacks of previous findings of others and new properties of the proposed ones that are clearly presented in the above table.

IV. CONCLUSION

In this paper we have presented various improvements seen in measures of similarity between the intuitionistic fuzzy sets in the last three decades that were found to be important in application point of view.

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