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STUDY OF A BULK QUEUING WITH NON RELIABLE SERVER

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ABSTRACT

In this paper, we study the optimal operation of a single removable and non-reliable server in a Markovian queuing system under steady-state conditions. The system is in idle state before the arrival of customer and after the arrival of customer it is in working state, it may breakdown and systems goes to vacation during that period. Here, there is bulk arrival of customer and bulk service provided to them.

Keywords— Bulk Arrival, Bulk Service, Interarrival Time, Exponentially Distributed

I INTRODUCTION

Assumptions:

(i) The arrival of customer are according to Poisson process with parameterλ.

(ii) Service time of customer is exponentially distributed with parameter $1/\mu$.

iii) As long as the server on, it will serve the customer

(iv) When it is working, it is assumed that breakdown can happen at any time, with Poisson rate ϕ_1.

(v) Whenever breakdown occurs it is repaired Immediately with repair rate ϕ_2 where repair rate are assumed to be exponentially distributed.

(vi) The server goes on vacation when there is nocustomer in the server and come back only when N Customer in the queue.

(vi) The server goes on vacation when there is no customer in the server and come back only when N customer in the queue.

Steady – State Result

Notations: - Let us suppose

$i = 0$,	Represents the server is in idle state.
$i = 1$,	Represents the server is on and in working condition.
$i = 2$	Represents the server is turned on and found to be broken.
$p_0(r)$	the steady state probability that there are r customers in the system and the customer is not in service when the server is turned off where $r = 0, 1, \ldots\ldots N-1$.

$p_1(r)$ the steady state Probability that there are r customers in the system when server is on and in working condition r = 1, 2……

$p_2(r)$ The steady state probability that there are r customers in the system when server is turned on but found to be broken down r = 1, 2...

The steady state equation for $p_0(r)$, $p_1(r)$, $p_2(r)$

$$\lambda P_0(0) = \mu P_1(1) \quad \ldots(1)$$

$$\lambda \sum_{m=1}^{r} \xi_m P_0(x) = \lambda \sum_{m=1}^{r} \xi_m (r-m) \quad 1 \le r \le N-1 \quad \ldots(2)$$

$$\left(\sum_{m=1}^{r}\xi_m\lambda + \phi_1 + \sum_{m=1}^{r}\xi_m \mu\right)P_1(1) = \mu\sum_{m=1}^{r-1}\xi_m P_1(2) + \phi_2 P_2(1) \quad \ldots(3)$$

$$\left(\sum_{m=1}^{r}\xi_m\lambda + \phi_1 + \sum_{m=1}^{r}\xi_m \mu\right)P_1(r) = \sum_{m=1}^{r-1}\xi_m \lambda P_1(r-m) + \mu \sum_{m=1}^{r-1}\xi_m P_1(r+1) + \phi_2 P_2(r), \quad r \le N-1 \quad \ldots(4)$$

$$\left(\sum_{m=1}^{N-1}\xi_m\lambda + \phi_1 + \sum_{m=1}^{N-1}\xi_m \mu\right)P_1(r) = \lambda\sum_{m=1}^{N-1}\xi_m P_1(r-1) + \mu \sum_{m=1}^{r}\xi_m P_1(r+1) + \phi_2 P_2(r), \quad r \ge N+1 \quad \ldots(5)$$

$$\left(\lambda\sum_{m=1}^{r}\xi_m \div \phi_2\right)P_2(1) = \phi_1 P_1(1) \quad i = 1,2,.. \quad \ldots(6)$$

$$\left(\lambda\sum_{m=1}^{r}\xi_m \div \phi_2\right)P_2(r) = \lambda \sum_{m=1}^{r-1}\xi_m P_2(r-m) + \phi_2 P_1(r) \quad \ldots(7)$$

Solving (1) – (7) recursively, we obtain analytic solution $P_i(n)$ (i = 0,1,2) as follows

$$P_1(r) = P_1(1) + \sum_{m=1}^{r}\xi_m \frac{\lambda}{\mu}\left[\left(1+\frac{\phi_1}{\phi_1+\phi_2}\right)P_1(r-1) + \frac{\phi_1}{\phi_1+\phi_2}\sum_{N=1}^{r-2}\left(\frac{\lambda}{\lambda+\phi_2}\right)^{r-N-1} P_1(N)\right] \quad 2 \le r \le N \quad \ldots(8)$$

$$P_1(r) = \sum_{m=1}^{r}\xi_m \frac{\lambda}{\mu}\left[\left(1+\frac{\phi_1}{\lambda+\phi_2}\right)P_1(r-1) + \frac{\phi_1}{\phi_1+\phi_2}\sum_{N=1}^{r-2}\left(\frac{\lambda}{\lambda+\phi_2}\right)^{r-N-1} P_1(N)\right] \quad r \ge N+1 \quad \ldots(9)$$

$$P_2(1) = \left[\frac{\phi_1}{\phi_1 + \phi_2}\right] P_1(1) \quad \ldots(10)$$

$$P_2(r) = \sum_{m=1}^{r}\xi_m \frac{\phi_1}{\lambda+\phi_2}\left[\sum_{N=1}^{r-1}\left(\frac{\lambda}{\lambda+\phi_2}\right)^{r-N} P_1(N) + P_1(r)\right], \quad r \ge 2 \quad \ldots(11)$$

Using equations (1) - (7)

$$[\lambda\sum_{m=1}^{r}\xi_m z^2-(\lambda\sum_{m=1}^{r}\xi_m+\phi_1+\sum_{m=1}^{r}\xi_m \mu)]Q_1(z)+\frac{\phi_2\phi_1}{\sum\xi_m\lambda+\phi_2-\sum\xi_m\lambda z}P_1(z) \quad = \sum_{m=1}^{r}\xi_m \,.\, z\,(1-z^N)\,P_0(0)$$

$$\ldots(12)$$

$$\phi_1 P_1(z) + (\sum_{m=1}^{r}\xi_m \sum \xi_m\lambda\, z - \sum_{m=1}^{r}\xi_m \sum\xi_m\lambda - \phi)\, P_2(z) = 0 \quad \ldots(13)$$

$$Q_2(z) = \frac{\psi}{\sum\xi_m\lambda+\phi-\sum\xi_m\lambda z}Q_1(z)$$

$$[\sum_{m=1}^{r}\xi_m\ \lambda z^2 -(\sum_{m=1}^{r}\xi_m\ \lambda+\alpha+\sum_{m=1}^{r}\xi_m\ \mu]\,P_1(z)+\phi(z).\frac{\psi}{\sum\xi_m\lambda+\phi-\sum\xi_m\lambda z}P_1(z) = \sum_{m=1}^{r}\xi_m\ \lambda z\,(1-z^N)P_0(0)$$

Let P(z) denote the generating function of the number of customers in the system, thus

$$P(z) = P_0(z) + P_1(z) + P_2(z)$$

$$P(z) = \left\{\frac{1-z^N}{1-z}+\frac{\sum\xi_m\lambda z(1-z^N)(\sum\xi_m\lambda z(1-z^N)-\phi-\psi)}{[\sum\xi_m\lambda z^2-(\sum\xi_m\lambda+\mu+\psi)z+\sum\xi_m\mu]\,[\sum\xi_m\lambda-\lambda-\psi]-\phi\psi z}\right\} \quad \ldots(14)$$

We can obtain $P_0(0)$ with the use of normalizing conditions P(z) = 1 i.e.

$$1 = \operatorname*{lt}_{z\to1} P(z)$$

$$P_0(0) = \frac{\sum\xi_m\mu\psi-\sum\xi_m\lambda(\phi+\psi)}{N\phi.\psi} \quad \ldots(15)$$

Computation for P_{ID}, P_{BS}, P_{BD}

In steady state,

P_{ID} : probability that server is idle.

P_{BS} : probability that server is busy.

P_{BD} : probability that server is breakdown

Let us suppose we have following expressions for P_{ID}, P_{BS}, and P_{BD}

$$P_{ID} = \sum_{r=0}^{N-1}P_0(r)$$

$$P_{BS} = \sum_{r=1}^{P}P_1(r)$$

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$$P_{BD} = \sum_{r=1}^{\infty} P_2(r)$$

It is clear from equation (2) that

$$P_{ID} = N\, P_0(0)$$

$$= N.\ \frac{\sum \xi_m.\mu\psi - \sum \xi_m \lambda(\phi+\psi)}{N \sum \xi_m \mu\psi}$$

$$P_{ID} = 1 - \frac{\sum \xi_m \lambda}{\sum \xi_m \mu\psi}(\psi + \phi)$$

$$=1\frac{\rho}{\psi}(\psi + \phi) \qquad \ldots(16)$$

If equations (1) – (6) added term by term

$$\phi P_{BS} = \psi\, P_{BD} \qquad \ldots(17)$$

since we have

$$P_{BS} + P_{BD} = 1 - P_{ID}$$

$$= 1 - \left(1 - \frac{\rho}{\psi}(\phi+\psi)\right)$$

$$= \frac{\rho}{\psi}(\phi + \psi)$$

then from equation (16), we obtain

$$P_{BD} = \frac{\phi}{\psi} P_{BS}$$

$$P_{BD} + \frac{\phi}{\psi} P_{BD} = 1 - P_{ID}$$

$$\left(\frac{\phi+\psi}{\phi}\right) P_{BD} = 1 - P_{ID}$$

$$P_{BD} = \rho$$

$$P_{BD} = \frac{\phi}{\psi} P_{BS} \qquad \ldots(18)$$

$$= \frac{\phi}{\psi}\rho. \qquad \ldots(19)$$

we prove that N-policy Markovian queuing system with a non-reliable server, the probability that the server is busy in steady state is equal to the traffic intensity. It is important to note that Heyman [4] has shown that the fraction of time server is busy for N-policy M/G/1 queuing system with reliable server is traffic intensity too.

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Special Cases

Case I :- If we take $\phi = 0$, $\psi = 0$, then the results for the N-policy M/M/1 queuing system with reliable server case are obtained. Expression for equation (13) for P (z) then corresponds to the existing result in the literature.

Case II: - If p we put $\phi = 0$ and N = 1 then result for ordinary M/M/1 queueing system with reliable server are obtained. Expression equation (13) for $P_0(0)$ then corresponds to the existing result in the literature.

System Characteristics

In steady-state, the following notations are used:

L_{IL} The expected number of customers in the system when the server is turned off;

L_{ID} The expected number of customers in the system when the server is turned on and is in operation;

L_{BD} The expected number of customers in the system when the server is turned on but found to be broken down;

L_S The expected number of customers in the system

The expressions for L_{IL}, L_{ID}, L_{BD} and L_S

$$L_{IL} = \sum_{r=1}^{N-1} r.P_0(r)$$

$$L_{ID} = \sum_{r=1}^{\infty} r.P_1(r)$$

$$L_{BD} = \sum_{r=1}^{\infty} r.P_2(r)$$

we have,

$$L_S = L_{IL} + L_{ID} + L_{BD}$$

To determine an expression for L_{IL}, we compute $P_0(z)$,

$$P_0(z) = P_0(0)\ \frac{(1-z^N)}{(1-z)}$$

$$P'_0(z) = \frac{(1-z)(-Nz^{N-1}) + (1-z^N)(-1)}{(1-z)^2}$$

$$= \frac{(N-1)N}{2}\frac{\left(\sum \xi_m \mu\psi - \sum \xi_m \lambda(\phi+\psi)\right)}{N\sum \xi_m \mu\psi}$$

$$= \frac{N-1}{2} - \frac{N+1}{2}\sum \xi_m\, \lambda \frac{(\phi+\psi)}{\sum \xi_m .\mu\psi}$$

$$L_{IS} = \frac{N-1}{2} - \rho\frac{(N-1)(\phi+\psi)}{2\beta} \qquad \ldots(20)$$

similarly L_{ID}, L_{BD}, L_S can be obtained by equations (9), (10), (11), we will compute first $P'_1(z)$, $P'_2(z)$ and $P'(z)$ then by applying L-Hospital rule twice, we will obtained.

$$L_{ID} = \frac{\rho(N+1)}{2} + \frac{\rho^2\left(\sum \xi_m \lambda\phi + \phi\psi + \psi^2\right)}{\psi[\psi - \rho(\phi+\psi)]} \qquad ...(21)$$

$$L_{BD} = \frac{\rho\phi(N+1)}{2\psi} - \frac{\rho^2\phi\left(\sum \xi_m \lambda - \phi - \psi - \sum \xi_m \lambda\right)}{\psi(\psi - \rho(\phi+\psi))} \qquad ...(22)$$

and

$$L_S = \frac{(N-1)}{2} + \frac{\rho\left(\sum \xi_m \lambda\phi + \phi\psi + \psi^2\right)}{\psi(\psi - P(\phi+\psi))} \qquad ...(23)$$

Note that the expected number of customers in the system for the ordinary M/M/1 queuing system with non-reliable server is obtained by setting N = 1 in (23).

Optimal N. Policy: - Let us defined the following

- ❖ When the server is turned off, then the length of time which is called Idle period is denoted by IL. Then the expected length is denoted by E[IL]
- ❖ Busy period to type I denoted by BS_1, which is length of time per cycle when server is turned on and in operation and type 1 customers are being served. The expected length of busy period type-I is $E[BS_1]$
- ❖ Busy period of type II denoted by BS_2 , which is the length of time per cycle when the server is turned on and in operation and type II customers are being served. Then expected length of busy period type-II is $E[BS_2]$
- ❖ Breakdown period denoted by BO which is the length of time per cycle when the server is turned on and found to be breakdown and customers are waiting to serve. Then expected length of breakdown is given by E[BD]
- ❖ Busy cycle denoted by BC, this is the length of time from the beginning of the last idle period to the beginning of the next idle period. Then expected length of busy cycle is given by E[BC].

We know since busy cycle is the sum of the idle period, the busy period of type-I, the busy period of type-II and the breakdown therefore we obtain

$$E[BC] = E[IL] + E[BS_1] + E[BS_2] + E[BD]$$
$$= E[L] + E[BS] + E[BD]$$

where $E[BS] = E[BS_1] + E[BS_2]$

using the memory less property for the exponential distribution, the length of the idle period is the sum of N exponential random variables each having mean $1/\lambda$. Thus the expected length of the idle period is given by

$$E[L] = \frac{N}{\lambda}$$

Determining the Optimal Policy

We construct a total expected cost function per unit time for the N-policy Markovian queueing system with non-reliable server, in which N is a decision variable our objective is to determine the optimum value of the control parameter N, say N*; so as to minimize this function. Let

C_h : be the holding cost per unit time per customer present in system

C_f :the cost incurred per unit time for keeping the server off

C_0 :the cost incurred per unit time for keeping the server on

C_b :the breakdown cost per unit item for failed server

C_s :the start-up cost per unit time for turning the server on

C_d :the shut down cost per unit time for turning the server off.

Using the definition of each cost element mentioned above, the total expected cost function per unit time is given by

$$F(R) = C_f E[R_S] + C_0 \frac{E[BS]}{E[CS]} + C_f \frac{E[L]}{E[CS]} + C_b \frac{E[BD]}{E[CS]} + (C_S + C_D)\frac{1}{E[CS]} \quad \ldots(24)$$

Since R is positive integer R = 1, 2,….., the optimal value of R, R*, to minimize F(R) is determined by satisfying inequality

$$F(R^*+1) > F(R^*) < F(R^*-1) \quad \ldots(25)$$

The inequality can be shown to reduces to

$$(R^*-1)R^* < \frac{2\lambda(C_s + C_d)[\psi - \rho(\phi+\psi)]}{C_h\,\psi} R^*(R^*+1) \quad \ldots(26)$$

As an approximation, if N is treated as a continuous variable greater than zero, then the optimum value of R is obtained by differentiating F(R) with respect to R and setting the result equal to zero. The optimal value of N is approximately given by

$$R^* \approx \sqrt{\frac{2\lambda(C_s + C_d)[\psi - \rho(\psi+\phi)]}{C_h\,\psi}} \quad \ldots(27)$$

It should be noted that if R* is not an integer, the best positive integers surrounding R*. It is be noted that the result equation (27) is more general that Heyman is result.

Numerical Results and Conclusion

The numerical results are obtained by considering the cost parameter as follows:

The optimal value of R policy and its minimum expected cost F(R) for the above five cases are shown in **Table-I** for $(\mu, \psi) = (2, 10)$ and for the variable values of (ψ, λ)

The following conclusions can be drawn from the table that

(i) F(R*) decreases as λ or ϕ decreases for any cases

(ii) R* decreases in λ for any case;

(iii) The optimum value of R, R*, increases as C_s and C_d increases for fixed value of R, C_k, C_0, C_f and C_b;

(iv) The optimum value of R, R* decreases as C_k increases for fixed value of C_s, C_0, C_f, C_d and C_b. From the last three columns of table-I, R* does not change even though ϕ varies from 0.1 to 0.3 for any case. Intuitively this seems too insensitive to changes in ϕ.

The optimal R policy and its minimum expected cost F(R) for five cases are shown in table-II for $(\lambda, \phi) = (0.5, 0.06)$ for different value of (μ, ψ).

The results in table-II clearly show that

F(R*) increases as μ or ψ decreases for any case.

(i) The optimum value of R, R* increases in μ for any case.

(ii) The optimum value of R, R* increases as C_s and C_d increases for fixed value of C_k, C_0, C_p and C_b; and

(iii) The optimum value of N, N* decreases as C_h increases for fixed values of C_s, C_0, C_f, C_d and C_b.

From the last three columns of table-II, R* are the same even thoughψ.

Table –I

φ	Ls	Ls	Ls	Ls
1.6	0.66	0.34	17.66	48.33
1.7	0.66	0.34	8.91	30.83
1.8	0.66	0.34	5.99	24.99
1.9	0.66	0.34	4.54	22.08
2	0.66	0.34	3.66	20.33
2.1	0.66	0.34	3.08	19.16
2.2	0.66	0.34	2.66	18.33
2.3	0.66	0.34	2.35	17.7
2.4	0.66	0.34	2.11	17.22
2.5	0.66	0.34	1.91	16.83
2.6	0.66	0.34	1.75	16.51
2.7	0.66	0.34	1.62	16.25
2.8	0.66	0.34	1.51	16.02
2.9	0.66	0.34	1.41	15.83

Graph-I

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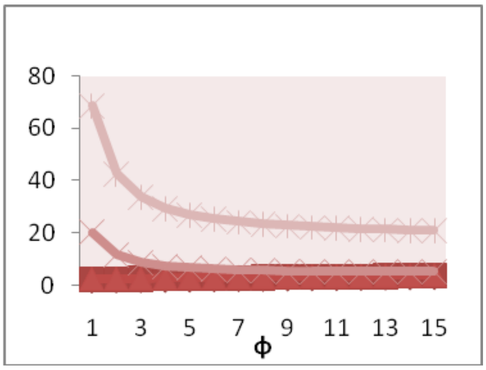


Table- II

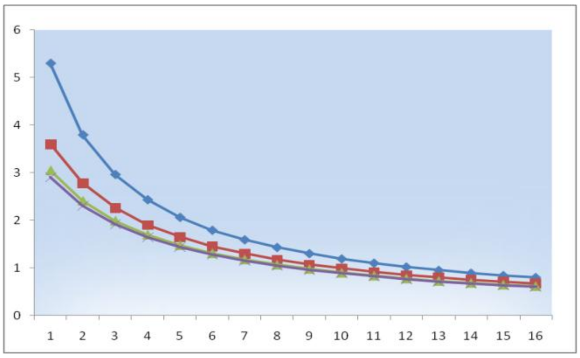
Values of expected Mean Queue length with respect to φ

μ = 3 μ = 3 μ = 3 μ = 3

λ = 9 λ = 11 λ = 13 λ = 15

φ	**Ls**	**Ls**	**Ls**	**Ls**
1.5	5.29	3.6	3.05	2.9
1.6	3.79	2.78	2.41	2.3
1.7	2.96	2.26	1.99	1.91
1.8	2.43	1.9	1.7	1.64
1.9	2.06	1.65	1.48	1.43
2	1.79	1.45	1.31	1.27
2.1	1.59	1.3	1.18	1.14
2.2	1.43	1.17	1.07	1.04
2.3	1.3	1.07	0.98	0.95
2.4	1.19	0.99	0.9	0.88
2.5	1.1	0.91	0.84	0.82
2.6	1.02	0.85	0.78	0.76
2.7	0.95	0.8	0.73	0.71
2.8	0.89	0.75	0.69	0.67
2.9	0.84	0.71	0.65	0.63
3	0.8	0.67	0.62	0.6

Graph-II



REFERENCES

1. Doshi B.T. , (1986) , Queueing system with vacation, A survey Queueing system vol. 1, pp. 29-66.

2. Chaudhary M.L. and Gupta U.C.(1997) Queue length & waiting time distribution of discrete time GI/Geom./1 Queueing system with early & late arrivals. Queueing system vol. 25 pp. 307-324.

3. Chaudhary M.L. Templeton, J.G.C. and Medhi (1992) Computational results of multi-server bulk arrival queues with constant service time $M^X/D/C$ Operation research vol. 40 pp. 229-238.

4. Clarke, A.B.(1978) A multi-server general service time queue with servers in series, math. Rept. No. 51.

5. Cox, D.R. (1955) The analysis of non-Monrovian stochastic processes by the inclusion of Supplementary variables, philosophical soc 1 pp. 441-443.

6. Cooper R.B. (1970) Queues served in cycle order waiting times. The bell Journal 49 pp. 399-413.

7. Cooper R.B. (1976) Introduction to queuing theory Edward Arnold Ltd.

8. Connolly, B.W. (1960) Queueing at a single serving point with group arrival J.R. statose. B-22, pp. 285-298.

9. Krishnamoorthy, A & Usha Kumari, P. V.(2000) A queueing system with single arrival bulk service and single departure mathematical & computer Modeling vol. 31, pp. 99-108.