MEAN SQUARED KERNEL INDUCED FUZZY POSSIBILISTIC C-MEANS: AN ANALYZING HIGH DIMENSIONAL DATABASE

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ABSTRACT
In order to find an available subgroups in real life databases this paper presents effective fuzzy c-means by incorporating the membership function of fuzzy c-means, the typicality of possibilistic c-means approaches, mean squared kernel induced distance. To show the effectiveness of the proposed method this paper successfully implements the proposed method with synthetic dataset and it shows the superiority of the proposed method through clustering accuracy.

Keywords: Data Analysis, Fuzzy C-Means, Mean Squared Distance

1. INTRODUCTION

There are many mathematical assisted methods recently have been applied to find an available subgroups in high dimensional data [11]. Very recently clustering techniques [1, 6, 8, 14] have been used for finding available subgroups in complex high dimensional data. Clustering[3] involves the task of dividing data points into homogeneous classes or clusters so that items in the same class are as similar as possible and items in different classes are as dissimilar as possible [7, 15]. The fuzzy c-means clustering (FCM) algorithm has recently been applied to clustering the high dimensional real life related databases [4]. Though fuzzy c-means [5] works well in clustering the noise free database, it has considerable drawbacks in clustering the database which have noise and outliers. Furthermore, results in the above said studies have been inconsistent to clearly differentiate the data objects into appropriate clusters, and none has given importance to reduce the computational time, and the selection of blindness prototypes by random manner. The blindness of random prototype initialization in existed fuzzy c-means leads the clustering process as a time consuming task. Hence, in order to cluster effectively the objects which have similar expression patterns in high dimensional databases into different available subgroups, this paper proposes effective normed kernel distance based fuzzy clustering algorithms in the combination of both fuzzy membership function and typicality of possibilistic c-means. The combination of Possibility with fuzzy clustering has been successfully implemented to cluster the unlabeled data of real life problems by many researches [9, 13]. Here the typicality values are constrained and the sum of the overall data points of typicalities to a cluster is one. The proposed objective function is enhanced by introducing normed kernel
induced distance to evaluate the relations between cluster prototypes and data objects. The kernel induced distance helps to have higher dimensional feature space from original pattern space in order to obtain strong membership for a cluster. The new novel approach has implemented with synthetic database.

The rest of this paper is organized as follows. In Section 2, this paper briefly the basic fuzzy c-means. Section 3 describes about Proposed Fuzzy C-Means algorithm. The experimental results on Synthetic Dataset are reported in Section 4. Finally, conclusion is presented in Section 5.

II. FUZZY C-MEAN ALGORITHM

Fuzzy C-Means discovers soft clusters where a particular point can belong to more than one cluster with certain membership. The objective of the algorithm is to find the subgroups into the data so that the similarity of data objects within each subgroup is very higher. The functional of the Fuzzy C-Means Algorithm has three independent variables, \( U \) the membership matrix, \( X \) the data space and the vector of prototypes \( V \). Fuzzy C-Means \([2]\) is based on minimization of the following objective function:

\[
J_{FCM}(U,V) = \sum_{k=1}^{n} \sum_{i=1}^{c} (u_{ik})^m \|x_k - v_i\|^2 ,
\]

Where \( m \) is the weighting parameter greater than 1, \( u_{ik} \) is the partition matrix. The partitioning and centre updating equations are:

\[
u_{ik} = \left( \sum_{j=1}^{c} \left( \frac{\|x_k - v_i\|^2}{\|x_k - v_j\|^2} \right)^{\frac{1}{m-1}} \right)^{-1} \sum_{k=1}^{n} (u_{ik})^m x_k \]

\[
v_i = \frac{\sum_{k=1}^{n} (u_{ik})^m x_k}{\sum_{k=1}^{n} (u_{ik})^m}
\]

The algorithm is composed of the following steps

**Step1:**

(i) Set the cluster centre

(ii) Set the fuzzification parameter

(iii) Set the random partition

**Step2:**

Obtain partition matrix \( U \), using

\[
u_{ik} = \left( \sum_{j=1}^{c} \left( \frac{\|x_k - v_i\|^2}{\|x_k - v_j\|^2} \right)^{\frac{1}{m-1}} \right)^{-1} \sum_{k=1}^{n} (u_{ik})^m x_k \]

**Step3:**
Update the cluster centre using

\[ v_i = \frac{\sum_{k=1}^{n} (u_{ik})^m x_k}{\sum_{k=1}^{n} (u_{ik})^m} \]

Step 4:
If \[ |U^{(t+1)} - U^{(t)}| < \epsilon \] then stop the algorithm or go to step 2.

III. MEAN SQUARED DISTANCE KERNEL FUNCTION BASED FUZZY POSSIBILISTIC C-MEANS [MSFPCM]

This paper incorporates fuzziness weighting exponent, the expression of possibilistic typical weighting exponent \( \tau \) and means squared kernel induced distance with the objective of proposed fuzzy c-means. The proposed objective function of Fuzzy Possibilistic C-Means is given by

\[
J(U,V) = 2\sum_{i=1}^{c} \sum_{k=1}^{n} \left( u_{ik}^\alpha + \tau_{ik}^\beta \right) \beta - K(x_i,v_j) \text{ where } K(x_i,v_j) = \frac{\|x_i - v_j\|^2}{n}
\]

The proposed partition matrix in an objective function satisfies the following conditions:

\[ 0 \leq u_{ik} \leq 1, \text{ for } 1 \leq i \leq c, 1 \leq k \leq n, \]
\[ 0 < \sum_{k=1}^{n} u_{ik} < n, \text{ for } 1 \leq i \leq c, \]
\[ \sum_{k=1}^{n} \tau_{ik} = 1 \text{ for } 1 \leq i \leq c. \tag{2} \]

\( n \) in (1) is number of objects, and \( \beta \) is parameter. The weighting exponents compute the amount of fuzziness in the resulting classification in order to obtain proper center of cluster.

Minimizing the proposed objective function in equation (1) with respect to \( u_{ik}, \tau_{ik}, \text{ and } v_i \), we have obtain a generalized membership equations \( u_{ik} \) and typicality \( \tau_{ik} \) for the iterative solution of an objective function. The general iterative Membership and typicality value updating equation is as:

\[ u_{ik} = \frac{\lambda^{1/\tau}}{\sum_{j=1}^{n} \left( 1 + \frac{1}{(\beta - 1)v_i} \right)^{\tau/\beta}} \]
\[ \tau_{ik} = \left( \frac{1}{\sum_{j=1}^{n} \left( 1 + \frac{1}{(\beta - 1)v_i} \right)^{\tau/\beta}} \right)^{1/\beta} \tag{3} \]
The general center updating equation is as:

$$v_i = \frac{\sum_{m} (\mu_{m}^{n} + \tau_{m}^{n}) k_{i}}{\sum_{m} (\mu_{m}^{n} + \tau_{m}^{n})}$$

(5)

IV. EXPERIMENTAL RESULTS ON ARTIFICIAL IMAGE

This subsection describes the experimental results on artificial image which is generated by random data given in Fig. 1. There are two algorithms used in this section, i.e., FPCM [12], and MSFPCM for showing the performance of proposed method. First experiment of this paper introduces the FPCM algorithm to an artificial image which is generated by random data in Fig. 1. The artificial image includes two classes is given in Fig. 1(a-b). The results of standard FPCM are given in Table 1 and in Fig. 2. Table 1 lists the memberships obtained for each object in final iteration of standard FPCM. Fig. 2 gives the clustering result of FPCM. The FPCM takes 28 iterations to termination condition.

Fig. 1: (a) 2D Artificial Dataset & (b) Image by 2D Artificial Dataset
Fig. 2: Result by FPCM

Table 1. Memberships of Final Iteration of Standard FPCM

<table>
<thead>
<tr>
<th>Membership for cluster 1</th>
<th>Belonging</th>
<th>Membership for cluster 2</th>
<th>Belonging</th>
<th>Object</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.444489</td>
<td>Cluster 2</td>
<td>0.555511</td>
<td>Cluster 2</td>
<td>1</td>
</tr>
<tr>
<td>0.505239</td>
<td>Clu...</td>
<td>0.494761</td>
<td>Cluster 1</td>
<td>2</td>
</tr>
<tr>
<td>0.417151</td>
<td>Cluster 2</td>
<td>0.582849</td>
<td>Cluster 2</td>
<td>3</td>
</tr>
<tr>
<td>0.465711</td>
<td>Cluster 2</td>
<td>0.534289</td>
<td>Cluster 2</td>
<td>4</td>
</tr>
<tr>
<td>0.415176</td>
<td>Cluster 2</td>
<td>0.584824</td>
<td>Cluster 2</td>
<td>5</td>
</tr>
<tr>
<td>0.501166</td>
<td>Cluster 1</td>
<td>0.498834</td>
<td>Cluster 1</td>
<td>6</td>
</tr>
<tr>
<td>0.574661</td>
<td>Cluster 2</td>
<td>0.425339</td>
<td>Cluster 2</td>
<td>7</td>
</tr>
<tr>
<td>0.416436</td>
<td>Cluster 2</td>
<td>0.583564</td>
<td>Cluster 2</td>
<td>8</td>
</tr>
<tr>
<td>0.633863</td>
<td>Cluster 1</td>
<td>0.366137</td>
<td>Cluster 1</td>
<td>9</td>
</tr>
<tr>
<td>0.501074</td>
<td>Cluster 1</td>
<td>0.498926</td>
<td>Cluster 1</td>
<td>11</td>
</tr>
<tr>
<td>0.597672</td>
<td>Cluster 1</td>
<td>0.402338</td>
<td>Cluster 1</td>
<td>12</td>
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<tr>
<td>0.600000</td>
<td>Cluster 1</td>
<td>0.400000</td>
<td>Cluster 1</td>
<td>13</td>
</tr>
<tr>
<td>0.537437</td>
<td>Cluster 1</td>
<td>0.462563</td>
<td>Cluster 1</td>
<td>14</td>
</tr>
<tr>
<td>0.623101</td>
<td>Cluster 1</td>
<td>0.376899</td>
<td>Cluster 1</td>
<td>15</td>
</tr>
</tbody>
</table>
Now this paper introduces the proposed MSFPCM to cluster the artificial image into two clusters in order to test its effect on performance. Fig. 3 shows the results of MSFPCM on synthetic image. It is observed from Fig. 3 that the MSFPCM reduces the misclassification in ordering the objects into two clusters based on the intensities of objects, and it achieves better memberships to the objects for a particular cluster than FPCM which are listed in Table 2. Table 2 compares clustering centres to the average value of points corresponding to each cluster, and it is found that the average value of points are almost close to the centres obtained by MSFPCM. The algorithm obtains the results after five iterations of the algorithm.

![Fig. 3 Image by MSFPCM](image)

**Table 2 Memberships of Final Iteration of MSFPCM**

<table>
<thead>
<tr>
<th>Membership for cluster 1</th>
<th>Belonging</th>
<th>Membership for cluster 2</th>
<th>Belonging</th>
<th>Object</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.997561</td>
<td>Cluster 1</td>
<td>0.002439</td>
<td>Cluster 1</td>
<td>1</td>
</tr>
<tr>
<td>0.984777</td>
<td>Cluster 1</td>
<td>0.015223</td>
<td>Cluster 1</td>
<td>2</td>
</tr>
<tr>
<td>0.970592</td>
<td>Cluster 1</td>
<td>0.029408</td>
<td>Cluster 1</td>
<td>3</td>
</tr>
<tr>
<td>0.927684</td>
<td>Cluster 1</td>
<td>0.072316</td>
<td>Cluster 1</td>
<td>4</td>
</tr>
<tr>
<td>0.979993</td>
<td>Cluster 1</td>
<td>0.020007</td>
<td>Cluster 1</td>
<td>5</td>
</tr>
<tr>
<td>0.999689</td>
<td>Cluster 1</td>
<td>0.000311</td>
<td>Cluster 1</td>
<td>6</td>
</tr>
<tr>
<td>0.010525</td>
<td>Cluster 2</td>
<td>0.989475</td>
<td>Cluster 2</td>
<td>7</td>
</tr>
<tr>
<td>0.969565</td>
<td>Cluster 1</td>
<td>0.030435</td>
<td>Cluster 1</td>
<td>8</td>
</tr>
</tbody>
</table>
This paper has shown that the proposed methods have been converged the termination value within few numbers of iterations, and FPCM has taken large number of iterations to converge the termination value from Table 3. In order to evaluate the effect of membership equation of proposed method in obtaining memberships to objects on clustering data into appropriate clusters, the resulted memberships of this experimental study on synthetic image given in Fig. 4 (a-b). It is observed from Fig. 4(a) that there is no wide difference between the memberships of the objects between the first and second clusters, it is because of poor distance measure of FPCM. From Fig. 4(b) it is observed that the proposed methods have wide difference in between the membership values for the objects for first and second clusters.

![Comparison of membership (a) by FPCM (b) by MSFPCM](image)

Finally Table 3 shows the comparison of the number of iteration, running time, and clustering accuracy during the experiment of FPCM, and MSFPCM on synthetic image. The FPCM takes 14 iterations to complete the experimental work on synthetic image for clustering it into two partitions, but the proposed method has taken less number of iterations to complete the algorithms. It is clear from the all above observations; the proposed
method gives better clustering results, clustering accuracy [10], and high memberships for clustering the data into two groups. Further the proposed methods require less running time, and less number of iterations to complete the experimental works.

Table 3. Comparison of Iteration Count, Running Time and clustering accuracy

<table>
<thead>
<tr>
<th></th>
<th>No. of Iterations</th>
<th>No. of clusters</th>
<th>Running Time</th>
<th>Clustering Accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>FPCM</td>
<td>21</td>
<td>2</td>
<td>28 Seconds</td>
<td>57%</td>
</tr>
<tr>
<td>MSFPCM</td>
<td>5</td>
<td>2</td>
<td>5 Seconds</td>
<td>98.5%</td>
</tr>
</tbody>
</table>

From the results on synthetic image, this paper proves the impact of proposed method via convergence speed of optimal centers, number of iterations, accuracy of clustering results and visual inspection of separation of clusters by the method, that the proposed method can have more capable to cluster the similar expression of genes in colon cancer database.

V. CONCLUSION

This paper has proposed effective clustering technique based on the membership function of fuzzy c-means, the typicality of possibilistic c-means approaches, and normed kernel induced distance, for finding subgroups in databases. In order to establish the effectiveness of the proposed method, this paper demonstrated experimental works on Synthetic dataset. This paper has reported the superiority of the proposed methods through cluster validation using silhouette accuracy, running time, number of iterations and well separated clusters.

REFERENCES