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FUZZY CONTRA D-CONTINUOUS FUNCTIONS AND FUZZY STRONGLY D-CLOSED SPACES

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ABSTRACT

In this paper we study a new class of some strong form of fuzzy contra D-continuous functions and fuzzy strongly D-closed and introduce some of its characterization in fuzzy topology

Key Words: Fuzzy Super Closure, Fuzzy Super Interior, Fuzzy Super Closed, Fuzzy Super Open Set, Fuzzy Continuity, Fuzzy Super Continuity.

I INTRODUCTION

Several generalization of Fuzzy Super open and super closed sets Let X be a nonempty set and I =[0,1]. A fuzzy set on X is a mapping from X to 1. The null fuzzy set 0 on X into I which assumes only the values 0 and the whole fuzzy set 1 is a mapping from X on to [0, 1] which takes the values 1 only .The union (resp. intersection) of family $\{A_\alpha : \alpha \in \wedge\}$ of fuzzy set of X is defined to be the mapping sup A_α (resp. inf A_α).A fuzzy set A of X is contained in a fuzzy set B of X if $A(x) \leq B(x)$ for each $x \in X$. A fuzzy point x_β in X is a fuzzy set defined by $x_\beta(y) = \beta$ for $y = x$ and $x(y) = 0$ for $y \neq x$, $\beta \in [0,1]$ and $y \in X$ .A fuzzy point x_β is said to be quasi-coincident with the fuzzy set A denoted by $x_\beta qA$ if and only if $\beta + A(x) > 1$. A fuzzy set A is quasi coincident with a fuzzy set B is denoted by AqB if and only if there exists a point $x \in X$ such that $A(x)+B(x)>1$. $A \leq B$ if and only if $\rceil AqB^c$.

A family τ of fuzzy set of X is called the fuzzy topology on X if 0 and 1 belongs to τ and τ is closed with respect to arbitrary union and finite intersection . The member of τ are called fuzzy open sets and their compliment are fuzzy closed sets. For a fuzzy set A of X the closure of A (denoted by cl(A)) is the intersection of all the fuzzy closed superset of A and the interior of A (denoted by int(A)) is the union of all fuzzy open subsets of A.

In this paper we use the aberrations; Fuzzy D-open(FDO), Fuzzy D-closed (FDC)

II PRELIMINARIES

Let X be a nonempty set and I=[0,1] A fuzzy set in X is a mapping from X in to I. The null fuzzy set 0 is the mapping from X in to I which assumes only the value 0 and the whole fuzzy set 1 is a mapping from X in to I

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which takes value I only. The union $\cup A_\alpha$ (resp. intersection $\cap A_\alpha$) of a family { A_α : $\propto \in \Lambda$} of fuzzy sets of X is defined to be the mapping Sup A_α (resp. Inf. A_α). A fuzzy set A of X is contained in a fuzzy set B of X denoted by A B if and only if $A(x) \leq B(x)$ for each $x \in X$. The complement A^C or 1-A of a fuzzy set A is defined by 1-A(x) for each x E X. A fuzzy point x in X is a fuzzy set defined by

$$x_\beta(y) = \begin{cases} \beta & \beta \in (0,1] \text{ for } y=x, y \in X \\ 0 & \text{otherwise} \end{cases}$$

where x and β are respectively called the support and value of x. A fuzzy point $x_\beta \in A$ if and only if $\beta \leq A(x)$. A fuzzy set A is the union of all fuzzy points which belongs to A. A fuzzy point $x_\beta \in A$ is said to be quasi-coincident with the fuzzy set A denoted by $x_\beta q A$ if and only if B + A(x) > 1. A fuzzy set A is quasi-coincident with a fuzzy set B denoted by AqB if and only if there exist $x \in X$ such that A(x)+B(x) > 1. $A \leq B$ if and only if $\rceil(AqB^C)$.

Let f :X → Y be a mapping. If A is a fuzzy set of X, then f(A) is a fuzzy set of X defined by

$$f(A)(y) = \begin{cases} \sup A(x),\ x \in f^{-1}(y) & \text{if } f^{-1}(y) \neq \phi \\ 0 & \text{otherwise} \end{cases}$$

If B is a fuzzy set of Y, then $f^{-1}(B)$ is a fuzzy set of X defined by $f^{-1}(B)(x) = B(f(x))$, for each $x \in X$.A family τ of fuzzy sets of X is called a fuzzy topology on X .if 0 and I belongs to τ and τ is closed with respect to arbitrary union and finite intersection. The members of τ are called fuzzy open sets and their complements are fuzzy closed sets. For a fuzzy set A, the closure of A (denoted by cl(A)) is the intersection of all fuzzy closed super sets of A and the interior of A (denoted by int(A)) is the union of all fuzzy open subsets of A. A fuzzy set A of a fuzzy topological space (X, τ) is called fuzzy generalized closed (fuzzy g-closed) if $cl(A) \leq G$

whenever $A \leq G$ and G is fuzzy open .The complement of a fuzzy g-closed set is called fuzzy g- open ii A^C is fuzzy g-closed . Every fuzzy closed (resp. fuzzy open) set is fuzzy g-closed (resp. fuzzy g-open) but its converse may not be true.

Definition 2.1. Let (X,τ) be a fuzzy topological space. A subset A of the space X is said to be

1. Fuzzy semi open if $A \leq int(cl(A))$ and fuzzy pre closed if $cl(int(A)) \leq A$.
2. Fuzzy semi open if $A \leq cl(int(A))$ and fuzzy semi closed if $int(cl(A)) \leq A$.
3. Fuzzy Regular open if A = int(cl(A)) and fuzzy Regular closed if A = cl(int(A)).

Definition 2.2.Let (X,τ) be a fuzzy topological space . A subset $A \subseteq X$ is said to be

1. fuzzy g-closed if cl(A) ≤ U whenever A ≤ U and Uis fuzzy open in X.
2. fuzzy ω-closed if cl(A) ≤ U whenever A ≤U and U is fuzzy Semi open in X.
3. fuzzy D-closed if pcl(A) ≤ Int(U) whenever A ≤ U and U is fuzzy ω-open in X. The complements of above mentioned sets are called their respective fuzzy open sets.

Definition 2.3. A function f : (X,τ) → (Y,σ) is called

1. fuzzy g-continuous if f^{-1}(V) is fuzzy g-closed in (X,τ) for every fuzzy closed set V in (Y,σ).
2. fuzzy ω-continuous if f^{-1}(V) is fuzzy ω-closed in (X,τ) for every fuzzy closed set V in (Y,σ).
3. fuzzy Perfectly continuous if f^{-1}(V) is fuzzy clopen in (X,τ) for every fuzzy open set V in (Y,σ).
4. fuzzy D-continuous if f^{-1}(V) is fuzzy D-closed in (X,τ) for every fuzzy closed set V in (Y,σ).
5. fuzzy D-irresolute if f^{-1}(V) is fuzzy D-closed in (X,τ) for every fuzzy D-closed set V in (Y,σ).
6. fuzzy strongly D-continuous if f -1(V) is fuzzy closed in (X,τ) for every fuzzy D-closed set V in (Y,σ).
7. fuzzy Pre-D-continuous if f^{-1}(V) is fuzzy D- closed in (X,τ) for every fuzzy pre-closed set V in (Y,σ).
8. fuzzy Perfectly D-continuous if f^{-1}(V) is fuzzy clopen in (X,τ) for every fuzzy D-closed set V in (Y,σ).
9. fuzzy Super continuous if f^{-1}(V) is fuzzy Regular open in (X,τ) for every fuzzy open set V in (Y,σ).
10. Fuzzy contra-continuous if f^{-1}(V) is fuzzy closed in (X,τ) for every fuzzy open set V in (Y,σ).
11 . Fuzzy contra pre-continuous if f^{-1}(V) is fuzzy pre closed in (X,τ) for every fuzzy open set V in (Y,σ).
12. Fuzzy contra g-continuous if f^{-1}(V) is fuzzy g-closed in (X,τ) for every open set V in (Y,σ).
13. Fuzzy contra semi-continuous if f^{-1}(V) is fuzzy semi closed in (X,τ) for every fuzzy open set V in (Y,σ).
14. RC-continuous if f^{-1}(V) is fuzzy Regular closed in (X,τ) for every fuzzy open set V in (Y,σ).
15. D-open if f(V) is fuzzy D-open in (Y,σ) for every fuzzy D-open set V in (X,τ).

Definition 2.4. A space (X,τ) is called;

1. A $T_{1/2}$ space if every fuzzy g-closed set is fuzzy closed.ω Space if every fuzzy ω-closed set is fuzzy closed.
3. A fuzzy D-T_S space if every fuzzy D-closed set is fuzzy closed.
4. A fuzzy D-$T_{1/2}$ space if every fuzzy D-closed set is fuzzy pre closed.

Theorem 2.5 Let (X,τ) be a fuzzy topological space.

1. A subset A of (X,τ) is fuzzy Regular open if and only if A is fuzzy open and fuzzy D-closed.
2. A subset A of (X,τ) is fuzzy open and fuzzy Regular closed then A is fuzzy D-closed.

Theorem 2.6 Every fuzzy closed set in a fuzzy topological space (X,τ) is fuzzy D-closed.

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III FUZZY CONTRA-D-CONTINUOUS FUNCTIONS

Definition 3.1:A function $f : (X,\tau) \rightarrow (Y,\sigma)$ is called fuzzy contra-D-continuous if $f^{-1}(V)$ is fuzzy D-open (resp. fuzzy D-closed) in (X,τ) for every fuzzy closed (resp. fuzzy open) set V in (Y,σ).

Definition 3.2: Let A be a subset of a fuzzy topological space (X, τ). The set $\cap\{U\ \tau\in / A < U\}$ is called the kernel of A [19] and is denoted by Ker(A).

Lemma 3.4 : The following properties hold for subsets A, B of a fuzzy space X :

1. $x\in$ Ker(A) if and only if $A \cap F\ \phi \neq$ for any $F\in C(X, x)$.
2. A < Ker(A) and A = Ker(A) if A is fuzzy open in X.
3. If A < B then Ker(A) <Ker (B)

Theorem 3.1: Every fuzzy contra-continuous function is a fuzzy contra-D-continuous function. **Proof:** Let $f : (X,\tau) \rightarrow (Y,\sigma)$ be a fuzzy function. Let V be a fuzzy open set in (Y,σ).Since f is fuzzy contra- continuous, $f^{-1}(V)$ is fuzzy closed in (X,τ).Hence by $f^{-1}(V)$ is fuzzy D-closed in (X, τ). Thus f is a fuzzy contra-D-continuous function. Converse of this theorem need not be true.

Remark 3.1: Fuzzy contra-D-continuous and fuzzy contra-g-continuous (resp. fuzzy contra- continuous, fuzzy contra-D-continuous, fuzzy contra pre-continuous, fuzzy contra semi- continuous) are independent concepts.

Remark 3.2: The composition of two fuzzy contra D-continuous functions need not be fuzzy contra D-continuous.

Theorem 3.2: The following are equivalent for a fuzzy function $f : (X, \tau) \rightarrow (Y, \sigma)$: Assume that FDO(X) (resp. FDC(X)) is closed under any union (resp. intersection)

1. f is fuzzy contra-D-continuous
2. The inverse image of a fuzzy closed set V of Y is fuzzy D-open
3. For each $x\in X$ and each $V\in C(Y, f(x))$, there exists $U\in DO(X, x)$ such that $f(U) \subseteq V$.
4. $f(D\text{-}cl(A)) \leq Ker (f(A))$ for every subset A of X.
5. $D\text{-}cl(f^{-1}(B)) \leq f^{-1} (Ker (B))$ for every subset B of Y.

Proof The implications (1) $\Rightarrow$ (2), (2) $\Rightarrow$ (3) are obvious.

(3) $\Rightarrow$ (2) Let V be any fuzzy closed set of Y and $x \in f^{-1}(V)$ (V). Then $f(x) \in V$ and there exists $U\ x\in DO(X, x)$ such that $f(U_X) \subset V$. Hence we obtain $f^{-1}(V)$ (V) $= \cup\{U_X : x \in f^{-1}(V)$ (V))and by assumption $f^{-1}(V)$ is fuzzy D open.

(2) $\Rightarrow$ (4) Let A be any subset of X. Suppose that $y \notin$ Ker (f(A)).Then by Lemma 3.4, there exists $V \in C (X, x)$ such that $f(A) \cap V = \phi$. Thus we have $A \cap f-1(V) = \phi$ and $D\text{-}cl(A) \cap f-1(V) = \phi$. Hence we obtain $f(D\text{-}cl(A)) \cap V = \phi$ and $y \notin f(D - cl(A))$.Thus $f(D\text{-}cl(A)) \leq Ker(f(A))$.

(4) $\Rightarrow$ (5): Let B be any subset of Y. By (4) and $f(D\text{-}cl(f^{-1}(B))) < Ker(f(f^{-1}(B))) < ker(B)$ and$D- cl(f^{-1}(B)) < f-1(Ker (B))$.

(5) ⇒ (1) :Let U be any fuzzy open set of Y. Then we have D–cl(f^{-1}(U)) < f^{-1}(Ker(U)) = f^{-1}(U) and D-cl(f^{-1}(U)) = f^{-1}(U). By assumption, f^{-1}(U) is fuzzy D-closed in X. Hence f is fuzzy contra-D-continuous.

Theorem 3.3: If f : (X, τ) → (Y, σ) is fuzzy D-irresolute (resp. fuzzy contra-D-continuous) and g : (Y, σ) → (Z, η) in fuzzy contra-D-continuous (resp. fuzzy continuous) then their composition gof : (X,τ) → (Z, η) is fuzzy contra-D-continuous.

Proof : Let U be any fuzzy open set in (Z, η). Since g is fuzzy contra-D-continuous (resp. fuzzy continuous) then g^{-1}(V) is fuzzy D-closed (resp. fuzzy open) in (Y, σ) and since f is fuzzy D- irresolute (resp. fuzzy contra D-continuous) then $f^{-1}(g^{-1}$(V)) is D-closed in (X, τ). Hence gof is fuzzy contra-D-continuous.

Theorem 3.4 If f : (X,τ) → (Y,σ) is fuzzy contra-continuous and g : (Y,σ) → (Z, η) is fuzzy continuous then their composition gof : (X, τ) → (Z, η) is fuzzy contra-D-continuous. **Proof:**Let U be any fuzzy open set in (Z, η).Since g is fuzzy continuous, g^{-1}(U) is fuzzy open in (Y,σ).Since f is fuzzy contra-continuous, $f^{-1}(g^{-1}$(U)) is fuzzy closed in (X, τ).Hence by theorem 2.6, $(gof)^{-1}$(U) is D-closed in (X,τ).Hence gof is fuzzy contra-D-continuous.

Theorem 3.5: If f : (X, τ) → (Y, σ) is fuzzy contra-continuous and fuzzy super-continuous and g : (Y, σ) → (Z, η) is fuzzy contra continuous then their composition gof : (X, τ) → (Z, η) is fuzzy contra-D-continuous.

Proof:Let U be any fuzzy open set in (Z, η).Since g is fuzzy contra-continuous, g^{-1}(U) is fuzzy closed in (Y, σ) and since f is fuzzy contra-continuous and super-continuous then $f^{-1}(g^{-1}$(U)) is both fuzzy open and fuzzy Regular closed in (X, τ).Then $(gof)^{-1}$(U) is fuzzy D-closed in (X, τ). Hence gof is fuzzy contra-D-continuous.

Theorem 3.7: Let (X,τ), (Y,σ) be any fuzzy topological spaces and (Y,σ) be fuzzy $T_{1/2}$ space (resp. fuzzy T_{ω}- space).Then the composition gof : (X, τ) → (Z, η) of fuzzy contra-D-continuous function f : (X, τ) → (Y, σ) and the fuzzy g-continuous (resp. fuzzy ω-continuous) function g : (Y, σ) → (Z, η) is fuzzy contra-D-continuous.

Proof: Let V be any fuzzy closed set in (Z, η). Since g is fuzzy g-continuous (resp. fuzzy ω- continuous), g^{-1}(V) is fuzzy g-closed (resp. fuzzy ω-closed) in (Y, σ) and (Y, σ) is fuzzy $T_{1/2}$ space (resp. fuzzy T_{ω}-space), hence g^{-1}(V) is fuzzy closed in (Y, σ). Since f is fuzzy contra-D- continuous, $f^{-1}(g^{-1}$(V)) is fuzzy D-open in (X, τ).Hence gof is fuzzy contra-D-continuous. **Theorem 3.8** :If f : (X, τ) → (Y, σ) is a surjective fuzzy D-open function and g : (Y, σ) → (Z, η) is a function such that gof : (X, τ) → (Z, η) is fuzzy contra-D-continuous then g is fuzzy contra- D-continuous.

Proof : Let V be any fuzzy closed subset of (Z, η). Since gof is fuzzy contra-D-continuous then $(gof)^{-1}(V) = f^{-1}(g^{-1}(V))$ is fuzzy D-open in (X, τ) and since f is surjective and fuzzy D-open, then $f(f^{-1}(g^{-1}(V))) = g^{-1}(V)$ is D-open in (Y, σ). Hence g is fuzzy contra-D-continuous. **Theorem 3.9:**If f : (X, τ) → (Y, σ) is strongly D-continuous and g : (Y, σ) → (Z, η) is fuzzy contra-D-continuous then gof : (X, τ) → (Z, η) is fuzzy contra-continuous. **Proof:** Let U be any open set in (Z, η). Since g is fuzzy contra-D-continuous, then $g^{-1}(U)$ is D-closed in (Y, σ). Since f is fuzzy strongly D-continuous, then $f^{-1}(g^{-1}(U)) = (gof)^{-1}(U)$ is closed in (X, τ). Hence gof is fuzzy contra-continuous.

Theorem 3.10:If f : (X, τ) → (Y, σ) is fuzzy pre-D-continuous and g : (Y, σ) → (Z, η) is fuzzy contra-pre-continuous then gof : (X, τ) → (Z, η) is fuzzy contra-D-continuous.

Proof: Let U be any fuzzy open set in (Z, η). Since g is fuzzy contra-pre-continuous, then $g^{-1}(U)$ is fuzzy pre-closed in (Y, σ) and since f is fuzzy pre-D-continuous, then $f^{-1}(g^{-1}(U)) = (gof)^{-1}(U)$ is fuzzy D-closed in (X, τ). Hence gof is fuzzy contra-D-continuous. **Theorem 3.11**:If f : (X, τ) → (Y, σ) is strongly-D-continuous and g : (Y, σ) → (Z, η) is fuzzy contra-D-continuous then gof : (X, τ) → (Z, η) is fuzzy contra-D-continuous.

Proof: Let U be any fuzzy open set in (Z, η). Since g is fuzzy contra-D-continuous, then $g^{-1}(U))$ is fuzzy D-closed in (Y, σ) and since f is fuzzy strongly-D-continuous, then $f^{-1}(g^{-1}(U)) = (gof)^{-1}(U)$ is fuzzy closed in (X, τ). then $(gof)^{-1}(U)$ is fuzzy D-closed in (X, τ). Hence gof is fuzzy contra-D-continuous.

Theorem 3.12:Let f : (X, τ) → (Y, σ) be surjective fuzzy D-irresolute and fuzzy D-open and g : (Y,σ) → (Z, η) be any function. Then gof : (X, τ) → (Z, η) is fuzzy contra-D-continuous if and only if g is fuzzy contra-D-continuous.

Proof: The 'if' part is easy to prove. To prove the 'only if' part, let V be any fuzzy closed set in (Z, η). Since gof is fuzzy contra-D-continuous, then $(gof)^{-1}(V)$ is D-open in (X, τ) and since f is fuzzy D-open surjection, then $f((gof)^{-1}(V)) = g^{-1}(V)$ is fuzzy D-open in (Y,σ). Hence g is fuzzy contra-D-continuous.
Theorem 3.13:Let f : (X,τ) → (Y,σ) be a fuzzy function and g : X →X × Y the fuzzy graph function given by g(x) = (x,f(x)) for every x∈ X. Then f is fuzzy contra-D-continuous if g is fuzzy contra-D-continuous.

Proof: Let V be a closed subset of Y. Then X × V is a closed subset of X ×Y. Since g is fuzzy contra-D-continuous, then $g^{-1}(X \times V)$ is a fuzzy D-open subset of X. Also $g^{-1}(X \times V) = f^{-1}(V)$.Hence f is fuzzy contra-D-continuous.

Theorem 3.14:If a function f : (X, τ) → (Y, σ) is fuzzy contra-D-continuous and Y is fuzzy Regular, then f is fuzzy D-continuous.

Proof: Let x be an arbitrary point of X and N be a fuzzy open set of Y containing f(x). Since Y is fuzzy Regular, there exists an open set U in Y containing f(x) such that cl(U) $\subseteq$ N. Since f is fuzzy contra-D-continuous, then there exists W $\in$ DO(X, x) such that f(W) $\subseteq$ cl(U). Then f(W)$\subseteq$ N. Hence by f is D-continuous.

Theorem 3.15: Every continuous and fuzzy RC-continuous function is fuzzy contra-D- continuous. **Proof:** Let f : (X, τ) → (Y, σ) be a function. Let U be an fuzzy open set in (Y, σ). Since f is fuzzy continuous and fuzzy RC continuous, f^{-1}(U) is fuzzy open and fuzzy Regular closed in (X,τ). Hence f is fuzzy contra-D-continuous.

Theorem 3.16: Every continuous and fuzzy contra-D-continuous (resp. fuzzy contra-continuous and fuzzy D-continuous) function is a fuzzy super-continuous (resp. fuzzy RC–continuous) function.

Proof: Let f : (X, τ) → (Y, σ) be a fuzzy function. Let U be an fuzzy open (resp. fuzzy closed) set in (Y, σ). Since f is fuzzy continuous and fuzzy contra-D-continuous (resp. fuzzy contra- continuous and fuzzy D-continuous),f^{-1}(U) is fuzzy open and fuzzy D-closed in (X, τ),then f^{-1}(U) is fuzzy Regular open in (X, τ). This shows that f is a fuzzy super continuous (resp. fuzzy RC-continuous) function.

Theorem 3.17:Let f : (X, τ) → (Y, σ) be a fuzzy function and X a fuzzy D-T_S space. Then the following are equivalent.

1. f is fuzzy contra-D-continuous.
2. 2. f is fuzzy contra-continuous

Proof:(1) ⇒ (2).:Let U be an open set in (Y, σ). Since f is fuzzy contra-D-continuous, f^{-1}(U) is D-closed in (X, τ) and since X is fuzzy D-Ts space, f^{-1}(U) is closed in (X, τ). Hence f is fuzzy contra continuous.

(2) ⇒ (1).:Let U be an open set in (Y, σ). Since f is fuzzy contra-continuous, f^{-1}(U) is fuzzy closed in (X, τ). Hence f^{-1}(U) is fuzzy D-closed in (X, τ). Hence f is fuzzy contra-D-continuous.

IV FUZZY CONTRA-D-CLOSED AND STRONGLY D-CLOSED

Definition 4.1: The graph G(f) of a fuzzy function f : (X, τ) → (Y, σ) is said to be fuzzy contra- D-closed in X × Y if for each (x, y) $\in$ (X × Y) – G(f) there exist U $\in$ FDO(X, x) and V $\in$ C(Y, y) such that (U × V) $\cap$G(f) = ϕ.

Lemma 4.1: The graph G(f) of a fuzzy function f : (X, τ) → (Y, σ) is fuzzy contra-D-closed if and only if for each (x, y) ∈ (X ×Y) – G(f), there exists U ∈ FDO(X, x) and V ∈ C(Y, y) such that f(U) ∩ V = ϕ.

Theorem 4.1: If f : (X, τ) → (Y, σ) is fuzzy contra-D-continuous and Y is Urysohn then G(f) is fuzzy contra-D-closed in X × Y.

Proof : Let (x, y) ∈ X × Y – G(f).Then y ≠ f(x) and there exist fuzzy open sets V,W such that f(x)∈ V, y ∈ W and cl(V) ∩ cl(W) = ϕ. Since f is fuzzy contra-D-continuous and by theorem 3.12 there exists U∈ DO(X, x) such that f(U) ≤ V. Hence f(U) ∩ cl(W) = ϕ.Thus G(f) is fuzzy contra D-closed in X × Y.

Definition 4.2: A topological space (X,τ) is said to be

1. fuzzy Strongly S-closed if every fuzzy closed cover of X has a finite sub cover.
2. fuzzy S-closed if every fuzzy Regular closed cover of X has a finite sub cover.
3. Strongly compact if every fuzzy Semi open cover of X has a finite sub cover.
4. fuzzy Locally indiscrete if every fuzzy open set of X is fuzzy closed in X.
5. fuzzy Midly Hausdorff if the fuzzy δ-closed sets form a network for its fuzzy topology τ, where a fuzzy δ-closed set is the intersection of fuzzy Regular closed sets.
6. fuzzy Ultra normal if each pair of non-empty disjoint fuzzy closed sets can be separated by disjoint fuzzy clopen sets
7. fuzzy Nearly compact if every fuzzy Regular open cover of X has a finite sub cover.
8. fuzzy D-compact if every fuzzy D-open cover of X has a finite sub cover.
9. fuzzy D-connected if X cannot be written as the disjoint union of two non-empty fuzzy D- open Sets.

Definition 4.3: A fuzzy topological space (X,τ) is said to be fuzzy strongly D-closed if every fuzzy D-closed cover of X has a finite sub cover.

Theorem 4.2: Let (X, τ) be D-T_S space. If f : (X, τ) → (Y, σ) has a fuzzy contra-D-closed graph, then the inverse image of a fuzzy strongly S-closed set K of Y is fuzzy closed in (X, τ).

Proof: Let K be a fuzzy strongly S-closed set of Y and x ∈ f^{-1}(K). For each k∈ K, (x, k) ∉ G(f). then there exist U_k∈ DO(X, x) and V_k∈ C(Y, k) such that f(U_k) ∩V_k = ϕ.Since{K ∩V_k : k ∈ K} is a closed cover of the fuzzy subspace K, there exists a finite subset K_0<K such that K <∪{V_k: k∈ K_0}.Set U = ∩ {U_k : k ∈ K_0}.
Then U is fuzzy open, since X is a fuzzy D-Ts space .Therefore f(U) ∩ K=ϕ and U ∩f^{-1}(K) = ϕ.This shows that f^{-1}(K) is fuzzy closed in (X, τ).

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Theorem 4.3: If a fuzzy space (X,τ) is fuzzy strongly D-closed then the space is fuzzy strongly S-closed.

Proof: obvious.

Theorem 4.4: Let f : (X, τ) → (Y, σ) be a fuzzy contra-D-continuous and fuzzy pre-closed surjection. If (X,τ) is a fuzzy D-Ts, then (X,τ) is a fuzzy locally indiscrete space.

Proof: Let U be any fuzzy open set in (Y, σ). Since f is fuzzy contra-D-continuous and (X, τ) is a fuzzy D-Ts space, $f^{-1}(U)$ is fuzzy closed in (X, τ). Since f is a fuzzy pre-closed surjection, then U is fuzzy pre-closed in (Y, σ). Therefore cl(U) = cl (int(U)) < U. Hence U is fuzzy closed in (Y,

σ). Thus (Y, σ) is a fuzzy locally indiscrete space.

Theorem 4.5: If every fuzzy closed subset of a space X is fuzzy D-open then the following are equivalent.

1. X is fuzzy S-closed
2. X is fuzzy strongly S-closed

Proof:(1) ⇒ (2):Let $\{V_\alpha : \alpha\in I\}$ be a fuzzy closed cover of X then $\{V_\alpha : \alpha\in I\}$ is a fuzzy Regular closed cover of

X. Since X is fuzzy S-closed, then we have a finite sub cover of X. Hence X is fuzzy strongly S- closed.

(2) ⇒ (1):Let $\{V_\alpha : \alpha\in I\}$ be a fuzzy Regular closed cover of X. Since every fuzzy Regular closed is fuzzy closed and X is fuzzy strongly S-closed, then we have a finite sub cover of X. Hence X is fuzzy S-closed.

Definition 4.4:A fuzzy topological space (X, τ) is said to be ;

1. Fuzzy D-Hausdorff if for each pair of distinct points x and y in X there exist disjoint fuzzy D-open sets U and V of x and y respectively.

2. Fuzzy D-Ultra Hausdorff if for each pair of distinct points x and y in X there exist disjoint fuzzy D-clopen sets U and V of x and y respectively.

Theorem 4.6: If f : (X, τ) → (Y, σ) is fuzzy contra-D-continuous injection, where Y is Urysohn then the topological space (X, τ) is a D-Hausdorff.

Proof : Let x_1 and x_2 be two distinct points of (X, τ). Suppose $y_1 = f(x_1)$ and $y_2 = f(x_2)$. Since f is injective and $x_1 \neq x_2$ then $y_1 \neq y_2$. Since the space Y is Urysohn, there exist fuzzy open sets V and W such that $y_1\in V$, $y_2\in W$ and cl(V)∩cl(W) = ϕ. Since f is fuzzy contra-D-continuous and , there exist fuzzy D-open sets $U_{x_1}\in FDO(X, x_1)$ and $U_{x_2}\in FDO(X,x_2)$ such that $f(U_{x_1}) < cl(V)$ and $f(U_{x_2}) < cl(W)$. Thus we have $U_{x_1}\cap U_{x_2} = \phi$, since cl(V)∩cl(W) = ϕ. Hence X is a fuzzy D- Hausdorff.

Theorem 4.7: If f : (X,τ) → (Y,σ) is a fuzzy contra-D-continuous injection, where Y is fuzzy D- ultra Hausdorff then the fuzzy topological space (X, τ) is fuzzy D-Hausdorff. **Proof**

Let x_1 and x_2 be two distinct points of (X, τ). Since f is injective and Y is fuzzy D-ultra Hausdorff, then $f(x_1) \neq f(x_2)$ and also there exist fuzzy clopen sets U and W of Y such that $f(x_1)\in U$ and $f(x_2)\in W$, where U∩W = ϕ. Since f is fuzzy contra-D-continuous, x_1 and x_2 belong to fuzzy D-open sets $f^{-1}(U)$and $f^{-1}(W)$ respectively, where $f^{-1}(U) \cap f^{-1}(W) = \phi$.Hence X is D- Hausdorff.

Lemma 4.15 : Every fuzzy mildly Hausdorff strongly S-closed space is fuzzy locally indiscrete. **Theorem**

4.8: If a fuzzy function f : (X, τ) → (Y, σ) is fuzzy continuous and (X, τ) is a fuzzy locally indiscrete space, then f is fuzzy contra-D continuous. **Proof:** Let U be any fuzzy open set in (Y, σ). Since f is fuzzy continuous, $f^{-1}(U)$ is fuzzy open in (X, τ) and since (X, τ) is fuzzy locally indiscrete, $f^{-1}(U)$ is fuzzy closed in (X, τ). Hence by theorem 2.6, $f^{-1}(U)$ is fuzzy D-closed in (X, τ). Thus f is fuzzy contra-D-continuous. **Lemma 4.2:** If a function f : (X, τ) → (Y, σ) is fuzzy continuous and (X, τ) is fuzzy mildly Hausdorff strongly S-closed space then f is fuzzy contra-D-continuous.

Proof: Obvious.

Theorem 4.9: A fuzzy contra-D-continuous image of a fuzzy D-connected space is connected. **Proof:** Let f : (X, τ) → (Y, σ) be a fuzzy contra-D-continuous function of fuzzy D-connected space onto a fuzzy topological space Y. If possible, assume that Y is not fuzzy connected. Then Y = A∪B, A ϕ≠, B ϕ≠ and A∩B = ϕ, where A and B are fuzzy clopen sets in Y. Since f is fuzzy contra-D-continuous = $f^{-1}(Y) = f^{-1}(A\cup B) = f^{-1}(A) \cup f^{-1}(B)$, where $f^{-1}(A)$ and $f^{-1}(B)$ are non- empty D-open sets in X. Also $f^{-1}(A) \cap f^{-1}(B) = \phi$. Hence X is not fuzzy D-connected, which is a fuzzy contradiction. Therefore Y is fuzzy connected.

Definition 4.5:A topological space (X, τ) is said to be fuzzy D-normal if each pair of non-empty disjoint closed sets can be separated by disjoint fuzzy D-open sets.

Theorem 4.10:If f : (X, τ) → (Y, σ) is a closed fuzzy contra-D-continuous injection and Y is fuzzy ultra-normal, then X is fuzzy D normal.

Proof: Let V_1 and V_2 be non-empty disjoint fuzzy closed subsets of X. Since f is fuzzy closed and injective, then $f(V_1)$ and $f(V_2)$ are non-empty disjoint fuzzy closed subsets of Y. Since Y is fuzzy ultra-normal, then $f(V_1)$ and $f(V_2)$ can be separated by disjoint fuzzy clopen sets W_1 and W_2 respectively. Hence$V_1 \subset f^{-1}(W1)$ and $V_2 \subset f^{-1}(W1)$.Since f is fuzzy contra-D-continuous, then $f^{-1}(W_1)$ and $f^{-1}(W_2)$ are fuzzy D-open subsets of X and $f^{-1}(W_1) \cap f^{-1}(W_2) = \phi$. Hence X is fuzzy D-normal.

Theorem 4.11: The image of a fuzzy strongly D-closed space under a fuzzy contra-D-continuous surjective function is fuzzy compact.

Proof: Suppose that f : (X, τ) → (Y, σ) is a fuzzy contra-D-continuous surjection. Let $\{V_\alpha : \alpha \in I\}$ be any fuzzy open cover of Y. Since f is fuzzy contra-D-continuous, then $\{f^{-1}(V_\alpha):\alpha\in I\}$ is a fuzzy D-closed cover of X. Since X is fuzzy strongly D-closed, then there exists a finite subset I_0 of I such that $X = \cup\{f^{-1}(V_\alpha) : \alpha\in I_0\}$. Thus we have $Y = \cup\{V_\alpha : \alpha\in I_0\}$. Hence Y is fuzzy compact.

Theorem 4.12: Every fuzzy strongly D-closed space (X, τ) is a fuzzy compact S-closed space. **Proof:** Let $\{V\alpha : \in\alpha\ I\}$ be a cover of X such that for every $\alpha\in I$, V_α is fuzzy open and fuzzy Regular closed due to assumption. Then each V_α is fuzzy D-closed in X. Since X is fuzzy strongly D-closed, there exists a finite subset I_0 of I such that $X = \cup\{V_\alpha : \alpha\in I_0\}$.Hence (X, τ) is a fuzzy compact S-closed space.

Theorem 4.13: The image of a fuzzy D-compact space under a fuzzy contra-D-continuous surjective function is fuzzy strongly S-closed.

Proof: Suppose that f : (X, τ) → (Y, σ) is a fuzzy contra-D-continuous surjection .Let $\{V_\alpha : \alpha\in I\}$ be any closed cover of Y. Since f is fuzzy contra-D-continuous, then $\{f^{-1}(V\alpha) :\in\alpha I\}$ is a D- open cover of X. Since X is fuzzy D-compact, there exists a finite subset I_0 of I such that X =
$\cup\{f^{-1}(V_\alpha) / \in\alpha I_0\}$. Thus we have $Y = \cup\{V_\alpha : \alpha\in I_0\}$.Hence Y is fuzzy strongly S-closed.

Theorem 4.14: The image of a fuzzy D-compact space in any D-Ts space under a fuzzy contra- D-continuous surjective function is fuzzy strongly D-closed.

Proof: Suppose that f : (X, τ) → (Y, σ) is a fuzzy contra-D-continuous surjection. Let $\{V_\alpha / \in\alpha I\}$ be any fuzzy D-closed cover of Y. Since Y is fuzzy D-Ts space, then $\{V_\alpha : \alpha\in I\}$ is a fuzzy closed cover of Y. Since f is fuzzy contra-D-continuous, then $\{f^{-1}(V_\alpha) : \alpha\in I\}$ is a fuzzy D-open cover of X. Since X is fuzzy D-compact, there exists a finite subset I0 of I such that X = ∪ {f
$^{-1}(V_\alpha) : \alpha\in I_0\}$. Thus we have $Y = \cup\{V_\alpha : \alpha\in I_0\}$. Hence Y is fuzzy strongly D-closed.

Theorem 4.15:The image of fuzzy strongly D-closed space under a fuzzy D-irresolute surjective function is fuzzy strongly D-closed.

Proof: Suppose that f : (X, τ) → (Y, σ) is an fuzzy D-irresolute surjection. Let $\{V_\alpha / \in\alpha I\}$ be any fuzzy D-closed cover of Y. Since f is fuzzy D-irresolute then $\{f^{-1}(V_\alpha) : \alpha\in I\}$ is a fuzzy D- closed cover of X. Since X is fuzzy strongly D-closed, then there exists a finite subset I_0 of I such that $X = \cup\{f^{-1}(V_\alpha) : \alpha\in I_0\}$. Thus, we have $Y = \cup\{V_\alpha : \alpha\in I_0\}$. Hence Y is fuzzy strongly D-closed.

Lemma 4.3: The product of two fuzzy D-open sets is fuzzy D-open.

Theorem 4.16: Let $f : (X_1, \tau) \to (Y, \sigma)$ and $g : (X_2, \tau) \to (Y, \sigma)$ be two fuzzy functions where Y is a fuzzy Urysohn space and f and g are fuzzy contra-D-continuous function. Then $\{(x_1,x_2) : f(x_1) = g(x_2)\}$ is fuzzy D-closed in the product space $X_1\times X_2$.

Proof: Let V denote the set $\{(x_1,x_2) : f(x_1) = g(x_2)\}$. In order to show that V is fuzzy D-closed, we show that $(X_1\times X_2) - V$ is fuzzy D-open. Let $(x_1,x_2) \notin V$. Then $f(x_1) \neq g(x_2)$.Since Y is Urysohn, there exist fuzzy open sets U_1 and U_2 $f(x_1)$ and $g(x_2)$ such that $cl(U_1)\cap cl(U_2) = \phi$. Since f and g are fuzzy contra-D-continuous, f $^{-1}(cl(U1))$ and $g^{-1}(cl(U2))$ are fuzzy D-open sets containing x_1 and x_2 in X_1 and X_2. Hence $f^{-1}(cl(U_1)) \times g^{-1}(cl(U2))$ is fuzzy D-open. Further $(x_1,x_2)\in f^{-1}(cl(U_1)) \times g^{-1}(cl(U2)) \subset ((X_1\times X_2) - V)$. If follows that $((X_1\times X_2) - V)$is fuzzy D-open. Thus V is fuzzy D closed in the product space $X_1 \times X_2$.

Lemma 4.4: If f : (X, τ) → (Y, σ) is fuzzy contra-D-continuous and Y is a fuzzy Urysohn space, then $V = \{(x_1, x_2) / f(x_1) = f(x_2)\}$ is fuzzy D-closed in the product space $X_1\times X_2$.

Theorem 4.17: Let f : (X, τ) → (Y, σ) be a fuzzy continuous function. Then f is fuzzy RC- continuous if and only if it is fuzzy contra-D continuous.

Proof: Suppose that f is fuzzy RC-continuous. Since every fuzzy RC-continuous function is fuzzy contra-continuous, Therefore f is fuzzy contra D-continuous. Conversely, Let V be any fuzzy open set in (Y, σ). Since f is fuzzy continuous and fuzzy contra-D-continuous, $f^{-1}(V)$ is fuzzy open and fuzzy D-closed in (X, τ) then

f^{-1}(V) is fuzzy Regular open in (X, τ). That is, int(cl(f^{-1}(V))) = f^{-1}(V). Since f^{-1}(V) is fuzzy open, int(cl(f^{-1}(V))) = int(f^{-1}(V)) and so cl(int(f^{-1}(V))) = f^{-1}(V). Therefore V is fuzzy Regular closed in (X, τ). Hence f is fuzzy RC-continuous.

Theorem 4.18: Let f : (X, τ) → (Y, σ) be fuzzy perfectly D-continuous function, X be fuzzy locally indiscrete space and connected. Then Y has a fuzzy indiscrete topology.

Proof:Suppose that there exists a proper fuzzy open set U of Y. Since Y is locally indiscrete, U is a fuzzy closed set of Y. Therefore U is a fuzzy D-closed set of Y. Since f is fuzzy perfectly D- continuous, f^{-1}(U) is a proper fuzzy clopen set of X. This shows that X is not fuzzy connected. Which is a fuzzy contradiction. Therefore Y has an indiscrete fuzzy topology.

Theorem 4.19: Let f : (X, τ) → (Y, σ) be a fuzzy contra-D-continuous function. Let A be a fuzzy open fuzzy D-closed subset of X and let B be an fuzzy open subset of Y. Assume that DC(X, τ) (the class of all fuzzy D-closed sets of (X, τ)) be fuzzy D-closed under finite intersections. Then, the restriction $f|_A$: (A, τ_A) → (B, σ_B) is a fuzzy contra-D-continuous function.

Proof: Let V be an fuzzy open set in (B, σ_B). Then V = B∩K for some fuzzy open set K in (Y, σ). Since B is an fuzzy open set of Y, V is an fuzzy open set in (Y, σ).By hypothesis f^{-1}(V) ∩ A = H_1 (say) is a fuzzy D-closed set in (X, τ). Since $(f|_A)^{-1}$ (V) = H_1, it is sufficient to show that H_1 is a fuzzy D-closed set in (A,τ_A). Let G_1 be fuzzy ω-open in (A,τ_A) such that $H_1 \subseteq G_1$. Then by hypothesis and G_1 is fuzzy ω-open in (X, τ). Since H_1 is a fuzzy D-closed set in (X, τ), we have pclX(H_1) ≤ int (G_1). Since A is fuzzy open and pclA(H_1) = pcl(X(H_1))∩ A ≤ int (G_1) ∩ int(A) = Int(G_1∩ A) ≤ Int(G_1) and so H_1 = $(f|_A)^{-1}$(V) is a fuzzy D-closed set in (A, τ_A). Hence $f|_A$ is fuzzy contra-D-continuous function.

Theorem 4.20: fuzzy topological space (X, τ) is nearly fuzzy compact if and only if it is fuzzy compact and fuzzy strongly D-closed .

Proof: Obvious .

Theorem 4.21: If a fuzzy topological space (X, τ) is locally indiscrete space then fuzzy compactness and fuzzy strongly D-closed are the same.

Proof: Let (X, τ) be a fuzzy compact space. Since (X, τ) is a locally indiscrete space, then every fuzzy open set is closed and fuzzy compactness and fuzzy strongly D-compactness are the same in a locally indiscrete fuzzy topological space.

Theorem 4.22: A fuzzy topological space (X, τ) is fuzzy S-closed if and only if it is fuzzy strongly S-closed and fuzzy D-compact.

Proof: Obvious.

REFERENCES

[1]. J. Antony Rex Rodrigo and K.Dass, A new type of generalized closed sets, Internat. J.Math. Archive -3(4),2012,1517 – 1523

[2]. J. Antony Rex Rodrigo and K.Dass, Weak and strong form of D-irresolute functions (Accepted).

[3]. J. Antony Rex Rodrigo and K.Dass, D-continuous functions (communicated).

[4]. S.P.Arya and R.Gupta,On strongly continuous mappings,Kyungpook Math.J.,131 -143.1974 [5]. K. Balachandran, P. Sundaram, H. Maki, On generalized continuous maps in topological spaces, Mem. Fac. Sci. Kochi Univ. Ser.A.Math 12 (1991),5-13.

[6]. M. Caldas, S. Jafari, T. Noiri, M. Simeos, A new generalization of fuzzy contra-fuzzy continuity Via Levines fuzzy g-closed sets, Chaos Solitons Fractals 42 (2007),1595-1603.

[7]. M. Caldas, S. Jafari, Some properties of fuzzy contra-β-continuous functions.Mem. Fac. Sci. Kochi Univ. Ser. A Math, 22 (2001), 19-28.

[8]. J. Dontchev, Fuzzy contra-continuous functions and strongly S-closed spaces, Internat. J.Math. Math. Sci. 19 (1996), 303-310.

[9]. J. Dontchev, S. Popvassilev, D. Stovrova, On the η-expansion topology for the co- semi. Fuzzy Regularization and midlyHausdorff spaces, Acta Math.Hungar. 80(1998),9-19.

[10]. J. Dontchev, T.Noiri, Fuzzy contra-semicontinuous functions, Math. Pannon.10(1999),159- 168

[11]. Y.Gnanambal, K. Balachandran, On gpr-continuous functions in topological spaces, Indian J. Pure Appl. Math. 30(6) (1999), 581 - 593.

[12]. S. Jafari, T.Noiri, Fuzzy contra-super-continuous functions, Ann. Univ. Sci. Budapest 42(1999), 27-34.

[13]. S. Jafari, T.Noiri, Fuzzy contra-α-continuous functions between topological spaces , Iran Int. J. Sci. 2 (2001), 153-167

[14]. S. Jafari, T.Noiri, On fuzzy contra-precontinuous functions , Bull. Malays.Math.Sci.Soc. 25(2) (2002), 115-128.

[15]. N. Levine, Semi-open sets, Semi-fuzzy continuity in topological spaces, Amer. Math. Monthly 70 (1963), 36-41.

[16]. N. Levine, Generalized closed sets in topology,Rend.Circ, Math. Palermo 19(2)(1970), 89-96.

[17]. A.S. Mashour, M.E. Abd El-Monsef, S.N. El-Deep, On pre continuous and weak Precontinuous mappings, Proc. Math. Phys. Soc.Egypt 53 (1982), 47-53.

[18]. A.S. Mashour, M.E. Abd El-Monsef, I.A.Hasanein, T. Noiri, Strongly compact spaces, Delta J.Sci. 8 (1984), 30-46.

[19]. M. Mrsevic, On pairwise Roand R1 bitopological spaces , Bull. Math. Soc. Sci.Math. R.S. Roumanie 30 (78) (1986), 141 -148.

[20]. T. Nieminen, On ultrapseudocompact and related spaces , Ann. Acad. Sci. Fenn. Ser. A I Math. 3 (1977), 185-205.

[21]. T.Noiri, Super-fuzzy continuity and some strong forms of fuzzy continuity, Indian J.Pure Appl. Math. 15 (1984), 241 -250.

[22]. T.M. Nour, contributions to the theory of bitopological spaces, Ph.D. Thesis, Univ. Of Delhi, 1989.

[23]. M. Sheik John, A Study on generalizations of closed sets and continuous maps in topological and bitopological spaces ,Ph.D.Thesis, Bharathiar university, Coimbatore (2002). [24]. M.K. Singal, Mathur, On nearly compact spaces, Boll. Un. Mat. Ital., Serie IV (4-6) (1969), 702-710.

[25]. R. Staum, The algebra of bounded continuous functions into a non-archimedean field, Pacific J.Math. 50 (1974), 169-185.

[26]. M.H. Stone, Application of the theory Boolean rings to general topology, Trans. Amer.Math. Soc.,41, 375 – 381. 1937.

[27]. P. Sundaram, M.Sheik John, Weakly closed sets and weak continuous functions in topological spaces , Proc. 82nd Indian Sci. Cong.
Calcutta, (1995), 49.

[28]. P. Sundaram, M.Sheik John, On ω-closed sets in topology, ActaCienc. IndicaMath. 4 (2000), 389-392.

[29]. T.Thompson, S-closed spaces Proc. Amer. Math. Soc. 60 (1976), 335-338. .