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NORMAL SHOCK WAVE DIFFRACTION BY YAWED WEDGES

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ABSTRACT

Diffraction of normal shock wave with yawed wedges have been considered in this paper. Pressure distribution over the diffracted shock has been obtained for Mach number of the shock wave equal to two and angle of yaw to 40^0.

Keywords : Normal shock wave, Diffraction, Yawed Wedge, Pressure Distribution

I. INTRODUCTION

Lighthill (1949) considered the diffraction of a normal shock wave past a small bend of angle δ. Chester (1954) extended the work of Lighthill (1949) to the case of yawed wedges i.e in the case of Chester (1954), leading edge of the wedge is not parallel to the normal shock front. In Lighthill (1949) case the flow is expending with respect to time while in Chester's (1954) case it could be regarded as growing with respect to the axis of cone of disturbance. Chester (1954) determined pressure distribution over the wedge surface while Srivastava (2007) determined the Vorticity distribution of a particle over the diffracted shock for varying angle of yaw and for the same numbers of the shock wave as were taken by Lighthill (1949). Earlier Srivastava (2003) has obtained the Vorticity distribution when the angle of yaw equals zero. In the present investigation we have obtained the pressure distribution over the diffracted shock for M=2 (M is the Mach number of the shock wave) and β=400 (β is the angle of yaw). Reference may be made to the book by Srivastava (1994).

II. MATHEMATICAL FORMULATION

If the velocity of the shock is U and wedge is yawed through an angle β, then the point of intersection of the shock front and leading wedge travels along the leading edge of the wedge with velocity $U/\sin\beta$. If an equal and opposite velocity is superimposed on the whole field, the shock becomes stationary and we have the steady flow behind the shock. The flow in fact in many respects has similarity to Busemann's (1943) cone field problem. In the introductory part we have indicated how the flow is developing both in Lighthill case (1949) and Chester's case (1954).

Behind the normal shock there will be region of uniform flow which is not affected by the presence of the wedge. In this region the fluid velocity, pressure, density and sound velocity are denoted by q_1, p_1, ρ_1 and a_1 respectively; ahead of the shock the corresponding quantities are q_0, p_0, ρ_0 and a_0. U being the velocity of the shock, the shock transition relations for $\gamma = 1.4$ (γ being the ratio of specific heats) give

$$q_1 = \frac{5}{6}U\left(1 - \frac{a_0^2}{U^2}\right)$$

$$p_1 = \frac{5}{6}\rho_0\left(U^2 - \frac{a_0^2}{7}\right) \qquad \text{- (1)}$$

$$\rho_1 = \frac{6\rho_0}{\left(1 + \frac{5a_0^2}{U^2}\right)}$$

Let $M = \frac{U}{a_0}$ be the Mach number of the shock and let $M_1 = \frac{q_1}{a_1}$ be the Mach number of the uniform flow behind the shock, then form (1)

$$M_1 = \frac{5(M^2 - 1)}{[(7M^2 - 1)(M^2 + 5)]^{1/2}} \qquad \text{- (2)}$$

As indicated earlier imposition of velocity $U/\sin\beta$ on the whole field in a direction opposite to the direction of motion of the point of intersection of the shock and leading edge, the shock becomes stationery. The resultant velocity behind the shock for stationary configuration say V_1 is

$$V_1^2 = q_1^2 + \frac{U^2}{\sin^2\beta} + 2q_1\frac{U}{\sin\beta}\cos(\beta + 90)$$

$$= q_1^2 + \frac{U^2}{\sin^2\beta} - 2Uq_1 \qquad \text{- (3)}$$

The velocity is supersonic provided

$$q_1^2 + \frac{U^2}{\sin^2\beta} - 2Uq_1 > a_1^2 \qquad \text{- (4)}$$

There is a restriction on β for different Mach numbers for V_1 to be supersonic.

Now it is assumed that the condition (4) is satisfied. The perturbations introduced by the presence of the wedge are confined to the region bounded by the shock front and Mach cone with the vertex at the junction of the shock and the wedge leading wedge. This axis of the Mach cone is in the direction of $\vec{V_1}$ and subtends an angle μ with the shock (see Figure-1 of Chester (1954)) where

$$\tan\mu = \frac{U - q_1}{U\cot\beta} = \frac{(M^2 + 5)\tan\beta}{6M^2} \qquad \text{- (5)}$$

In may be mentioned here that for the relation (3), it would be helpful to see the Figure-1 of Chester (1954). Using small perturbation theory and using conical field transformation as used by Chester (1954), we obtain a single partial differential equation in *p*. The equation in *p* is

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = \left(x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y} + 1\right)\left(x\frac{\partial p}{\partial x} + y\frac{\partial p}{\partial y}\right) \qquad \text{- (6)}$$

In (6) $p = \dfrac{p_2 - p_1}{a_1 \rho_1 q_1}$, p_2 is the disturbed pressure and other quantities have already been defined, *x*, *y* are the transformed coordinates.

The characteristics of the differential equation (6) are tangents to the unit circle $x^2 + y^2 = 1$ which in the $Ox'y'z'$ system is the cone $x'^2 y'^2 = z'^2 \tan^2 \alpha$ Chester (1954). The region of disturbance will therefore be bounded by the Mach cone, wedge surface and the diffracted shock. If we take a section normal to the axis of Mach cone, the disturbed region will be represented by the arc of the circle $x^2 + y^2 = 1$, the wedge portion by the straight segment and the shock front by an arc in the (*x*, *y*) system of coordinates.

In the final transformed plane Chester (1954) gives

$$\frac{\partial p}{\partial y_1} + i\frac{\partial p}{\partial x_1} = \frac{-C\delta[D(z_1 - x_0) - 1]}{(z_1^2 - 1)^{1/2}(z_1 - x_0)\left[\gamma_1 - i(z_1 - 1)^{1/2}\right]\left[\gamma_2 - i(z_1 - 1)^{1/2}\right]} \qquad - (7)$$

$z_1 = x_1 + iy_1$ and on the real axis we have $z_1 = x_1$.

In order to find out the pressure over the diffracted shock, we have to obtain the imaginary part on the right hand side of (7) and equate it to the imaginary part of (7) on the left hand side. It one does that we have

$$\frac{\partial p}{\partial x_1} = \frac{-C\delta[D(x_1 - x_0) - 1](\gamma_1 + \gamma_2)(x_1 - 1)^{1/2}}{(x_1^2 - 1)^{1/2}(x_1 - x_0)\left[\gamma_1^2 + (x_1 - 1)\right]\left[\gamma_2^2 + (x_1 - 1)\right]} \qquad - (8)$$

On the shock we have relation

$$\frac{y}{k'} = \left(\frac{x_1 - 1}{x_1 + 1}\right)^{1/2} \qquad - (9)$$

In the transformed plane shock extends from $x_1 = 1$ to $x_1 = \infty$, when $x_1 = 1$, $y/k' = 0$ and when $x_1 \to \infty$, $y/k' = 1$. Integration of (8) from $x_1 = 1$ to $x_1 = \infty$, with breakup intervals gives the pressure distribution over the diffracted shock.

III. NUMERICAL CALCULATIONS

The numerical calculations have been carried out for M=2 and β=40^0. The following table show the results of the calculation:

Table

M=2 and β=40^0

y/k'	0	0.2	0.4	0.6	0.8	1
p	5.28	3.17	1.56	0.49	0.12	0

The table shows the value of p is highest at $y/k' = 0$ i.e at the point of intersection of shock and wall surface. Then it falls and finally approaches zero at $y/k' = 1$ i.e at point of intersection of shock and unit circle. The results are quite consistent.

IV. CONCLUSION

Diffraction of normal shock wave with yawed wedges are for more complicated than the case of diffraction of normal shock wave when the angle of yaw is zero. This is due to the fact that in the case of yawed wedges, the problem becomes three dimensional than what was in the case of angle of yaw being zero where the problem is two dimensional. These problems have a bearing in aeronautical engineering and will be useful in designing also.

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