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APPLICATION OF INTEGRAL TRANSFORM TECHNIQUE IN DETERMINATION OF THERMAL STRESSES WITH INTERNAL MOVING HEAT SOURCE

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ABSTRACT

Present paper deals with the study of thermal stresses in thin rectangular plate subjected to point heat source which changes its place along y axis. Governing heat conduction equation has been solved by using integral transform technique. Results are obtained in the form of infinite series. As a special case, aluminum plate has been considered and results for thermal stresses have been computed numerically and graphically.

Key Words: Rectangular plate, Internal moving point heat source, thermal stresses

I. INTRODUCTION

Material properties are dependent of change in temperature. The property like elasticity, stresses at various temperatures has been studied. These non-isothermal problems of theory of elasticity have attracted the attention of many. The temperature dependent properties are focused in various field like aerodynamics heating which produces intense thermal stresses reducing the strength of structure of high velocity aircraft [1].W.Nowaki[2] has determined the steady state thermal stresses in a circular plate subjected to an axisymmetric temperature distribution on the upper surface with zero temperature on the lower surface and with the circular edge thermally insulated.Boley and Weiner [3] have determined the thermal deflection of an axisymmetrically heated circular plate in the case of fixed and simply supported edges.Quasi static thermal stresses in a thin circular plate due to transient temperature taken along the circumference of a circle on the upper face with the lower face at zero temperature and a fixed circular edge thermally insulated have been considered by Roy Choudhary[4]. N.W.Khobragade and P.C.Wankhede [5] studied an inverse unsteady state thermoelastic problem of a thin rectangular plate. K.R.Gaikwadet al.[6] have been calculated quasi static thermoelastic problem of an Infinitely Long Circular Cylinder. Vijay B.Patil et al. [7] have calculated temperature distribution, thermal functions and displacement at any point of semi infinite Rectangular slab with internal heat source using integral transform technique.D.T.Solankeet al. [8] have been calculated temperature distribution and thermal stresses in thin rectangular plate with moving line heat source taking second kind boundary condition by using integral transform technique and green's theorem . M.S.Thakare et al.[9] have considered the problem on rectangular plate and find thermal stresses by integral transform with taking internal moving point heat source. Recently K.R.Gaikwad[10] has been determined temperature distribution, displacement and thermal stresses of a circular plate due to uniform internal energy generation using Hankel transform technique also illustrated graphically.Here is an attempt to determine effective solution and study of thermal stresses in a thin rectangular plate with internally moving heat point source.Present paper elaborates on determination of Temperature and thermal stresses in a thin rectangular plate defined as $0\le x\le a$, $0\le y\le b$,$-h\le z\le h$ where $h<b<a$, h is thickness which is very small. The governing heat conduction equation has been solved by using integral transform technique. Results are obtained in the form of infinite series. These have been computed numerically and graphically.

II. FORMULATION OF THE PROBLEM

We consider three dimensional thin rectangular plate under steady state temperature defined in region R: $0\le x\le a,0\le y\le b,-h\le z\le h$, where $h<b<a$, h is thickness which is very small. The plate is subjected to the motion of moving point heat source at the point$(0,y',0)$.Under these realistic prescribed conditions, temperature and thermal stresses in a thin rectangular plate are required to be determined.

Mathematical formulation of this problem is given by

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}+\frac{g}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{1}$$

Where k is thermal conductivity and α is thermal diffusivity of the material of the plate.

Consider an instantaneous moving heat source at point $(0, y^I, 0)$ and releasing its heat spontaneously at time t' . Such volumetric moving heat source in rectangular coordinates is given by

$$g(x,y,z,t)=g_p^i\,\delta(x)\,\delta(y-y')\,\delta(z)\,\delta(t-t^I) \tag{2}$$

Where g_p^i is instantaneous point heat source.

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}+\frac{1}{k}g_p^i\,\delta(x)\,\delta(y-y^I)\delta(z)=\frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{3}$$

With initial and boundary conditions are given by

$$[T]_{t=0}=0 \tag{4}$$

$$[T]_{x=a}=0 \tag{5}$$

$$[T]_{x=0}=0 \tag{6}$$

$$\left[\frac{\partial T}{\partial y}\right]_{y=b}=0 \tag{7}$$

$$\left[\frac{\partial T}{\partial y}\right]_{y=0}=0 \tag{8}$$

$$\left[T + K_1 \frac{\partial T}{\partial z}\right]_{z=h} = 0 \tag{9}$$

$$\left[T + K_2 \frac{\partial T}{\partial z}\right]_{z=-h} = 0 \tag{10}$$

Thermal stress function χ is $\chi = \chi_c + \chi_p$ where $\chi_c$complementary function is and χ_p is particular integral.χ_cAndχ_p are governed by an equations,

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)^2 \chi_c = 0 \text{ And} \tag{11}$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)^2 \chi_p = -\alpha E\Gamma \tag{12}$$

Plate is thin, z is negligible. And where $\Gamma = T - T_0$WhereT_0 is initial temperature. Also component of stress functions are given by

$$\sigma_{xx} = \frac{\partial^2 \chi}{\partial y^2} \tag{13}$$

$$\sigma_{yy} = \frac{\partial^2 \chi}{\partial x^2} \tag{14}$$

$$\sigma_{xy} = -\frac{\partial^2 \chi}{\partial x \partial y} \tag{15}$$

With boundary conditions

$\sigma_{yy} = 0\,, \sigma_{xy} = 0 \;\; at \;\; y = b$

Equations (1) to (15) represent the statement of the problem.

III. SOLUTION OF THE PROBLEM:

Applying Finite Fourier sine transform , finite Fourier cosine transform and marchi Fasulo transform also using boundary conditions, we get,

$$\frac{d\bar{\bar{T}}^*}{dt} + \alpha Q \bar{\bar{T}}^* = \emptyset \tag{16}$$

Where $Q = \frac{m^2\pi^2}{a^2} + \frac{n^2\pi^2}{b^2} + a_l{}^2$

$$\emptyset = \frac{g_p^i}{k} cos\left(\frac{n\pi y^l}{b}\right) p_l(0)\delta(t - t^l)$$

From (16) we get

$$\bar{\bar{T}}^* = (e^{-\alpha Q t} - 1)\frac{g_p^i}{k} cos\left(\frac{n\pi y^l}{b}\right) p_l(0)\, e^{\alpha Q t^l} \tag{17}$$

Taking inverse Marchi –fasulo Transform, Finite Fourier cosine transform and finite Fourier sine transform,

$$T = \frac{4}{ab}\sum_{l,m,n=1}^{\infty}\frac{p_l(z)}{\lambda_l}\left\{\left[\left(e^{-\alpha Qt}-1\right)\frac{g_p^i}{k}\cos\left(\frac{n\pi y'}{b}\right)p_l(0)\,e^{\alpha Qt'}\right]\cos\left(\frac{n\pi y}{b}\right)\right\}\sin\left(\frac{m\pi x}{a}\right) \quad (18)$$

and $\Gamma = T - T_0$

$$\Gamma = \frac{4}{ab}\sum_{l,m,n=0}^{\infty}\frac{p_l(z)}{\lambda_l}\left\{\left[\left(e^{-\alpha Qt}-1\right)\frac{g_p^i}{k}\cos\left(\frac{n\pi y'}{b}\right)p_l(0)\,e^{\alpha Qt'}\right]\cos\left(\frac{n\pi y}{b}\right)\right\}\sin\left(\frac{m\pi x}{a}\right) \quad (19)$$

IV. DETERMINATION OF STRESS FUNCTION

$$\chi_c = \sum_{m=1}^{\infty} y\left[c_1 e^{\frac{m\pi y}{a}} + c_2 e^{-\frac{m\pi y}{a}}\right]\cos\left(\frac{m\pi x}{a}\right) + y\left[c_3 e^{\frac{m\pi y}{a}} + c_4 e^{-\frac{m\pi y}{a}}\right]\sin\left(\frac{m\pi x}{a}\right) \quad (20)$$

$$\chi_p = \frac{4\alpha E ab}{\pi^2(a^2n^2+b^2m^2)}\sum_{l,m,n=0}^{\infty}\frac{p_l(z)}{\lambda_l}\left\{\left[\left(e^{-\alpha Qt}-1\right)\frac{g_p^i}{k}\cos\left(\frac{n\pi y'}{b}\right)p_l(0)\,e^{\alpha Qt'}\right]\cos\left(\frac{n\pi y}{b}\right)\right\}\sin\left(\frac{m\pi x}{a}\right) \quad (21)$$

$$\chi = \sum_{m=1}^{\infty} y\left[c_1 e^{\frac{m\pi y}{a}} + c_2 e^{-\frac{m\pi y}{a}}\right]\cos\left(\frac{m\pi x}{a}\right) + y\left[c_3 e^{\frac{m\pi y}{a}} + c_4 e^{-\frac{m\pi y}{a}}\right]\sin\left(\frac{m\pi x}{a}\right) + \frac{4\alpha E ab}{\pi^2(a^2n^2+b^2m^2)}\sum_{l,m,n=0}^{\infty}\frac{p_l(z)}{\lambda_l}\left\{\left[e^{-\alpha Qt}\frac{g_p^i}{k}\cos\left(\frac{n\pi y'}{b}\right)p_l(0)\,e^{\alpha Qt'}\right]\cos\left(\frac{n\pi y}{b}\right)\right\}\sin\left(\frac{m\pi x}{a}\right) \quad (22)$$

Using (22) in (13),(14) ,(15) we get,

$$\begin{aligned}\sigma_{xx} = \sum_{m=1}^{\infty}&\left\{\left[2\left(\frac{m\pi}{a}c_1 e^{\frac{m\pi y}{a}} - \frac{m\pi}{a}c_2 e^{-\frac{m\pi y}{a}}\right) + y\left(c_1 e^{\frac{m\pi y}{a}} + c_2 e^{-\frac{m\pi y}{a}}\right)\right]\cos\left(\frac{m\pi x}{a}\right)\right.\\ &\left.+\left[2\left(\frac{m\pi}{a}c_3 e^{\frac{m\pi y}{a}} - \frac{m\pi}{a}c_4 e^{-\frac{m\pi y}{a}}\right) + y\left(c_3 e^{\frac{m\pi y}{a}} + c_4 e^{-\frac{m\pi y}{a}}\right)\right]\sin\left(\frac{m\pi x}{a}\right)\right\}\\ &+\frac{4\alpha E ab}{\pi^2(a^2n^2+b^2m^2)}\sum_{l,m,n=0}^{\infty}\frac{p_l(z)}{\lambda_l}\left\{\left[\left(e^{-\alpha Qt}\right.\right.\right.\\ &\left.\left.\left.-1\right)\frac{g_p^i}{k}\cos\left(\frac{n\pi y'}{b}\right)p_l(0)\,e^{\alpha Qt'}\right]\cos\left(\frac{n\pi y}{b}\right)\right\}\sin\left(\frac{m\pi x}{a}\right)\left(\frac{n^2\pi^2}{b^2}\right)\end{aligned}$$

$$\begin{aligned}\sigma_{yy} = \frac{-m^2\pi^2 y}{a^2}&\left\{\left[c_1 e^{\frac{m\pi y}{a}} + c_2 e^{-\frac{m\pi y}{a}}\right]\cos\left(\frac{m\pi x}{a}\right) + \left[c_3 e^{\frac{m\pi y}{a}} + c_4 e^{-\frac{m\pi y}{a}}\right]\sin\left(\frac{m\pi x}{a}\right)\right\}\\ &-\frac{m^2\pi^2}{a^2}\frac{4\alpha E ab}{\pi^2(a^2n^2+b^2m^2)}\sum_{l,m,n=0}^{\infty}\frac{p_l(z)}{\lambda_l}\left\{\left[\left(e^{-\alpha Qt}\right.\right.\right.\\ &\left.\left.\left.-1\right)\frac{g_p^i}{k}\cos\left(\frac{n\pi y'}{b}\right)p_l(0)\,e^{\alpha Qt'}\right]\cos\left(\frac{n\pi y}{b}\right)\right\}\sin\left(\frac{m\pi x}{a}\right)\end{aligned}$$

$$\sigma_{xy} = \left\{\left[\left(c_1 e^{\frac{m\pi y}{a}} + c_2 e^{-\frac{m\pi y}{a}}\right) + \frac{m\pi y}{a}\left(c_1 e^{\frac{m\pi y}{a}} - c_2 e^{-\frac{m\pi y}{a}}\right)\right]\left(-\frac{m\pi}{a}\right)\sin\left(\frac{m\pi x}{a}\right) - \left[\left(c_3 e^{\frac{m\pi y}{a}} + c_4 e^{-\frac{m\pi y}{a}}\right) + \frac{m\pi y}{a}\left(c_3 e^{\frac{m\pi y}{a}} - c_4 e^{-\frac{m\pi y}{a}}\right)\right]\right\}$$

$$\left(-\frac{m\pi}{a}\right)\cos\left(\frac{m\pi x}{a}\right) + \frac{nm\pi^2}{ab}\frac{4\alpha Eab}{\pi^2(a^2n^2+b^2m^2)}\sum_{l,m,n=0}^{\infty}\frac{p_l(z)}{\lambda_l}\left\{\left[\left(e^{-\alpha Qt} - 1\right)\frac{g_p^i}{k}\cos\left(\frac{n\pi y^l}{b}\right)p_l(0)e^{\alpha Qt^l}\right]\sin\left(\frac{n\pi y}{b}\right)\right\}\cos\left(\frac{m\pi x}{a}\right)$$

Using the boundary condition $\sigma_{yy} = 0$ and $\sigma_{xy} = 0$ at $y = b$ we get,

$$c_1 = 0,\ c_2 = 0,\quad c_3 = \frac{2\alpha Eb\varphi(-a+m\pi b)}{\pi^2(a^2n^2+b^2m^2)}e^{\frac{-m\pi b}{a}},\quad c_4 = \frac{2\alpha Eb\varphi(a+m\pi b)}{\pi^2(a^2n^2+b^2m^2)}e^{\frac{m\pi b}{a}}$$

$$\sigma_{xx} = \sum_{m=1}^{\infty}\left\{\left[\left(\frac{2m\pi}{a}+y\right)\frac{2\alpha Eb\varphi(-a+m\pi b)}{\pi^2(a^2n^2+b^2m^2)}e^{\frac{-m\pi b}{a}}e^{\frac{m\pi y}{a}} + \left(-\frac{2m\pi}{a}+y\right)\frac{2\alpha Eb\varphi(a+m\pi b)}{\pi^2(a^2n^2+b^2m^2)}e^{\frac{m\pi b}{a}}e^{-\frac{m\pi y}{a}}\right]\sin\left(\frac{m\pi x}{a}\right)\right\} + \frac{4\alpha Eab}{\pi^2(a^2n^2+b^2m^2)}\sum_{l,m,n=0}^{\infty}\frac{p_l(z)}{\lambda_l}\left\{\left[\left(e^{-\alpha Qt} - 1\right)\frac{g_p^i}{k}\cos\left(\frac{n\pi y^l}{b}\right)p_l(0)e^{\alpha Qt^l}\right]\cos\left(\frac{n\pi y}{b}\right)\right\}\sin\left(\frac{m\pi x}{a}\right)\left(\frac{n^2\pi^2}{b^2}\right)$$

$$\sigma_{yy} = \frac{-m^2\pi^2 y}{a^2}\left\{\left[\frac{2\alpha Eb\varphi(-a+m\pi b)}{\pi^2(a^2n^2+b^2m^2)}e^{\frac{-m\pi b}{a}}e^{\frac{m\pi y}{a}} + \frac{2\alpha Eb\varphi(a+m\pi b)}{\pi^2(a^2n^2+b^2m^2)}e^{\frac{m\pi b}{a}}e^{-\frac{m\pi y}{a}}\right]\sin\left(\frac{m\pi x}{a}\right)\right\} - \frac{m^2\pi^2}{a^2}\frac{4\alpha Eab}{\pi^2(a^2n^2+b^2m^2)}\sum_{l,m,n=0}^{\infty}\frac{p_l(z)}{\lambda_l}\left\{\left[\left(e^{-\alpha Qt} - 1\right)\frac{g_p^i}{k}\cos\left(\frac{n\pi y^l}{b}\right)p_l(0)e^{\alpha Qt^l}\right]\cos\left(\frac{n\pi y}{b}\right)\right\}\sin\left(\frac{m\pi x}{a}\right)$$

$$\sigma_{xy} = \left\{\left[\left(-\left(1+\frac{m\pi y}{a}\right)\frac{2\alpha Eb\varphi(-a+m\pi b)}{\pi^2(a^2n^2+b^2m^2)}e^{\frac{-m\pi b}{a}}e^{\frac{m\pi y}{a}} - \left(1-\frac{m\pi y}{a}\right)\frac{2\alpha Eb\varphi(a+m\pi b)}{\pi^2(a^2n^2+b^2m^2)}e^{\frac{m\pi b}{a}}e^{-\frac{m\pi y}{a}}\right)\right]\left(-\frac{m\pi}{a}\right)\cos\left(\frac{m\pi x}{a}\right)\right\}$$

$$+\frac{nm\pi^2}{ab}\frac{4\alpha Eab}{\pi^2(a^2n^2+b^2m^2)}\sum_{l,m,n=0}^{\infty}\frac{p_l(z)}{\lambda_l}\left\{\left[\left(e^{-\alpha Qt} - 1\right)\frac{g_p^i}{k}\cos\left(\frac{n\pi y^l}{b}\right)p_l(0)e^{\alpha Qt^l}\right]\sin\left(\frac{n\pi y}{b}\right)\right\}\cos\left(\frac{m\pi x}{a}\right)$$

V. NUMERICAL RESULTS

Let k=0.5330, $\alpha = 23.8 \times 10^{-6}$, $E = 0.675 \times 10^{11}$, a=5cm, b=1 cm, h=0.2 cm

$$\sigma_{xx} = \sum_{m=1}^{\infty}\left\{\left[\left(\frac{2\pi}{5}+y\right)\frac{2\times 23.8\times 10^{-6}\times 0.675\times 10^{11}\times \varphi(-5+\pi)}{26\pi^2}e^{\frac{-\pi}{5}}e^{\frac{\pi y}{5}} + \left(-\frac{2\pi}{5}+y\right)\frac{2\times 23.8\times 10^{-6}\times 0.675\times 10^{11}\times \varphi(5+\pi)}{26\pi^2}e^{\frac{\pi}{5}}e^{-\frac{\pi y}{5}}\right]sin\left(\frac{\pi x}{5}\right)\right\}$$
$$+\frac{20\times 23.8\times 10^{-6}\times 0.675\times 10^{11}}{26\pi^2}\sum_{l,m,n=0}^{\infty}\frac{p_l(0.2)}{\lambda_l}\left\{\left[\left(e^{-23.8\times 10^{-6}Qt} - 1\right)\frac{g_p^i}{k}cos(\pi y^I)p_l(0)e^{\alpha Q t^I}\right]cos(\pi y)\right\}sin\left(\frac{\pi x}{a}\right)\left(\frac{n^2\pi^2}{b^2}\right)$$

$$\sigma_{yy} = \frac{-\pi^2 y}{25}\left\{\left[\frac{2\times 23.8\times 10^{-6}\times 0.675\times 10^{11}\times \varphi(-5+\pi)}{26\pi^2}e^{\frac{-\pi}{5}}e^{\frac{\pi y}{5}} + \frac{2\times 23.8\times 10^{-6}\times 0.675\times 10^{11}\times \varphi(5+\pi)}{26\pi^2}e^{\frac{\pi}{5}}e^{-\frac{\pi y}{5}}\right]sin\left(\frac{\pi x}{5}\right)\right\}$$
$$-\frac{\pi^2}{25}\frac{20\times 23.8\times 10^{-6}\times 0.675\times 10^{11}}{26\pi^2}\frac{p_l(0.2)}{\lambda_l}\left\{\left[\left(e^{-23.8\times 10^{-6}Qt} - 1\right)\frac{g_p^i}{k}cos(\pi y^I)p_l(0)e^{23.8\times 10^{-6}Qt^I}\right]cos(\pi y)\right\}sin\left(\frac{\pi x}{5}\right)$$

$$\sigma_{xy} = \left\{\left[\left(-\left(1+\frac{\pi y}{5}\right)\frac{2\times 23.8\times 10^{-6}\times 0.675\times 10^{11}\times \varphi(-5+\pi)}{26\pi^2}e^{\frac{-\pi}{5}}e^{\frac{\pi y}{5}} - \left(1-\frac{\pi y}{5}\right)\frac{2\times 23.8\times 10^{-6}\times 0.675\times 10^{11}\times \varphi(5+\pi)}{26\pi^2}e^{\frac{\pi}{5}}e^{-\frac{\pi y}{5}}\right)\right]\left(-\frac{\pi}{5}\right)cos\left(\frac{\pi x}{5}\right)\right\}$$

$$+\frac{\pi^2}{5}\frac{20\times 23.8\times 10^{-6}\times 0.675\times 10^{11}}{26\pi^2}\frac{p_l(0.2)}{\lambda_l}\left\{\left[\left(e^{-23.8\times 10^{-6}Qt} - 1\right)\frac{g_p^i}{k}cos(\pi y^I)p_l(0)e^{23.8\times 10^{-6}Qt^I}\right]sin(\pi y)\right\}cos\left(\frac{\pi x}{5}\right)$$

$$\varphi = \frac{p_l(0.2)}{\lambda_l}\left(e^{-23.8\times 10^{-6}Qt} - 1\right)\frac{g_p^i}{k}cos(\pi y^I)p_l(0)e^{\alpha Q t^I}$$

Where $y^I = 2$, $g_p^i = 1$, $t = 1$, $t^I = 1.5$ $\lambda_l = 15.5485$, $p_l(0.2) = 2.3562$ $\varphi = 0.6698e^{0.5\times 2.4796\times 10^{-4}} = 0.6699$ $Q = 10.4186$

$$T = \frac{4}{5}\times\frac{2.3562}{15.5485}\left\{\left[e^{-23.8\times 10^{-6}\times 10.4186t}\frac{1}{0.5330}(2.3562)e^{23.8\times 10^{-6}\times 10.4186(1.5)}\right]cos(\pi y)\right\}sin\left(\frac{\pi x}{5}\right)$$

$$\boldsymbol{\sigma_{yy} = \left\{\left[-3.2832\times 10^3 y e^{\frac{\pi y}{5}} + 4.9228\times 10^6 e^{-\frac{\pi y}{5}}\right]sin\left(\frac{\pi x}{5}\right)\right\} - 5.1750\times 10^5\left(e^{-2.4796\times 10^{-4}t} - 1\right)cos(\pi y)sin\left(\frac{\pi x}{5}\right)}$$

$$\boldsymbol{\sigma_{xx} = \sum_{m=1}^{\infty}\left\{\left[\left(-1.0450\times 10^4 - 8.3161\times 10^3 y\right)e^{\frac{\pi y}{5}} + \left(-1.5669\times 10^7 + 1.2469\times 10^7 y\right)e^{-\frac{\pi y}{5}}\right] + 1.2935\times 10^7\left[e^{-2.4796\times 10^{-4}t} - 1\right]cos(\pi y)\right\}sin\left(\frac{\pi x}{5}\right)}$$

$$\sigma_{xy} = \left\{\left[\left(\left(8.3161\times 10^3 + 5.2250\times 10^3 y\right)e^{0.6283y} - 7.8343\times 10^6 - 4.9223\times 10^6 y\right)e^{-0.6283y}\right)cos\left(\frac{\pi x}{5}\right)$$

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$$+1.0907\times10^{10}\left\{\left[e^{-2.4796\times10^{-4}t}-1\right]sin(\pi y)\right\}\cos\left(\frac{\pi x}{5}\right)$$

VI. GRAPHICAL INTERPRETATION

$$T=0.1212\{[(e^{-2.4796\times10^{-4}t}-1)(69.0869)]cos(\pi y)\}sin\left(\frac{\pi x}{5}\right)$$

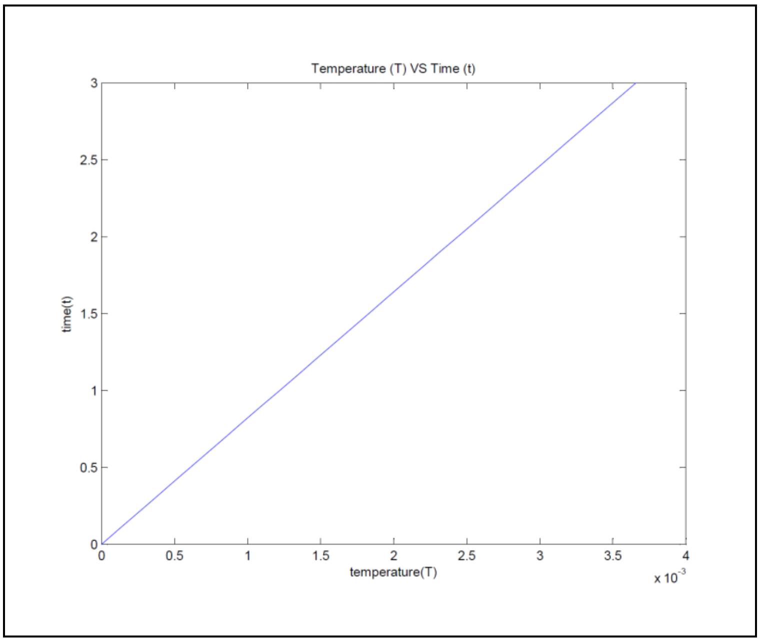

Figure 1: temperature Vs time

From Figure 1,In a rectangular plate(aluminium plate) observed that initial time made as constant and as we increase the time in certain region temperature also increases .

$$\sigma_{yy}=\left\{\left[-3.2832\times10^{3}ye^{\frac{\pi y}{5}}+4.9228\times10^{6}e^{-\frac{\pi y}{5}}\right]sin\left(\frac{\pi x}{5}\right)\right\}-5.1750\times10^{5}(e^{-2.4796\times10^{-4}t})cos(\pi y)sin\left(\frac{\pi x}{5}\right)$$

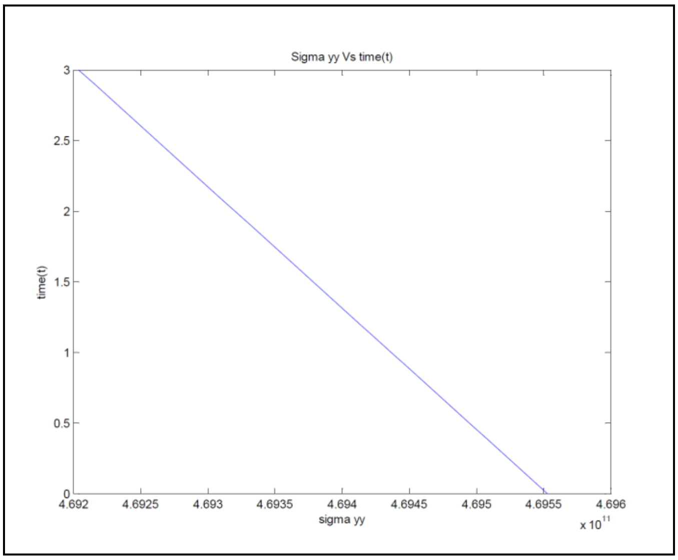

Figure2: sigmayyVstime

From figure2,Inaluminum plate when time is maximum thermal stresses high, as time reduced thermal stresses decreases.

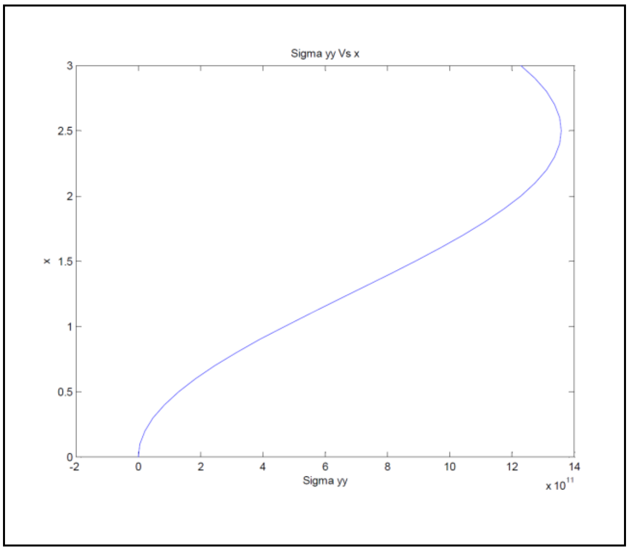

Figure3: sigma yyVs x

From figure 3,as value of x increase thermal stresses are also increased upto certain limit i.e. when value of x=2.5 it is maximum again it decrease as value of x increases.

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$$\sigma_{xx} = \left\{\left[\left(-1.0450\times10^{4} - 8.3161\times10^{3}y\right)e^{\frac{\pi y}{5}} + \left(-1.5669\times10^{7} + 1.2469\times10^{7}y\right)e^{-\frac{\pi y}{5}}\right] + 1.2935\times10^{7}\left[e^{-2.4796\times10^{-4}t}\right]cos(\pi y)\right\} sin\left(\frac{\pi x}{5}\right)$$

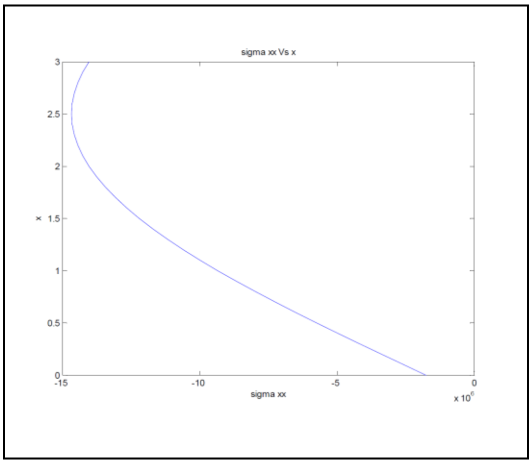

Figure 4: Sigma xx Vs x

From Figure 4, it has been seen that along x axis initially value of thermal stresses is quite large ,and it decrease as the value of x upto certain limit of x i.e. x=2.5 value of thermal stresses is very low again that value increase with value of x .

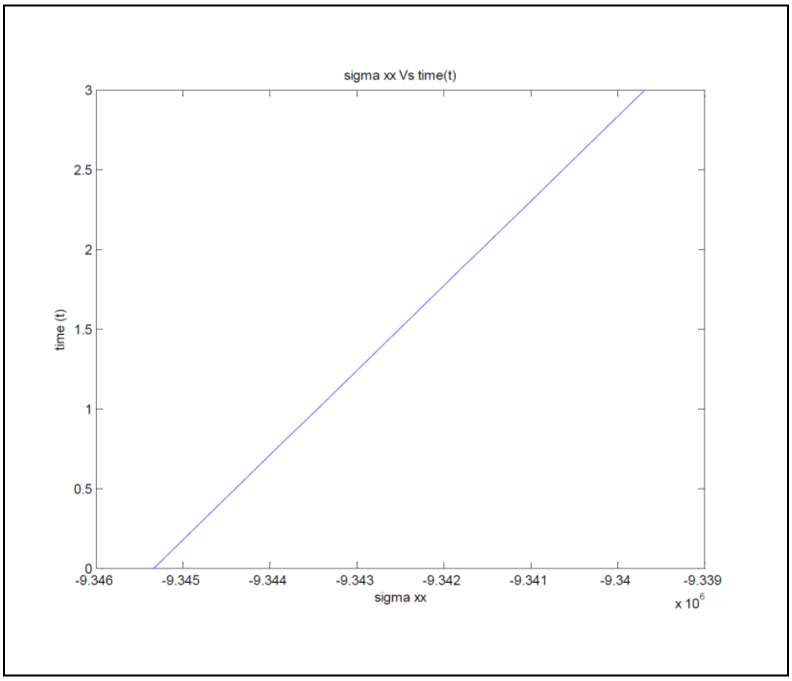

Figure5: sigma xx Vstime (t)

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From figure 5, thermal stress is directly proportionate with value of time i.e. if value of time increase value of thermal stresses also increases.

$$\sigma_{xy}=\{[((8.3161\times10^{3}+5.2250\times10^{3}y)e^{0.6283y}-(-7.8343\times10^{6}-4.9223\times10^{6}y)e^{-0.6283y})]\cos\left(\frac{\pi x}{5}\right)\}$$

$$+1.0907\times10^{10}\left\{\left[e^{-2.4796\times10^{-4}t}\right]\sin(\pi y)\right\}\cos\left(\frac{\pi x}{5}\right)$$

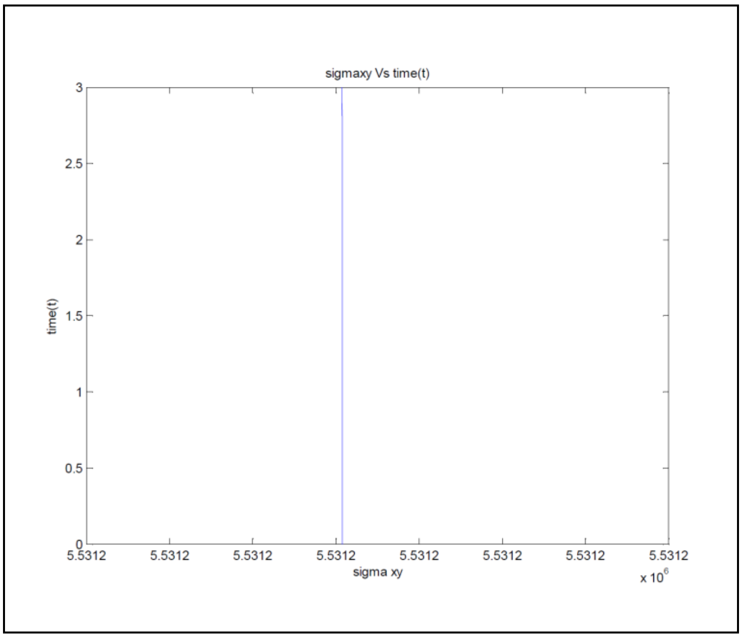

Figure 6: sigma xyVstime(t)

From figure 6, stresses is constant along xy while change in value of time. In aluminum plate there is small change in y axis plate slightly changing its position along y axis so stresses are constant.

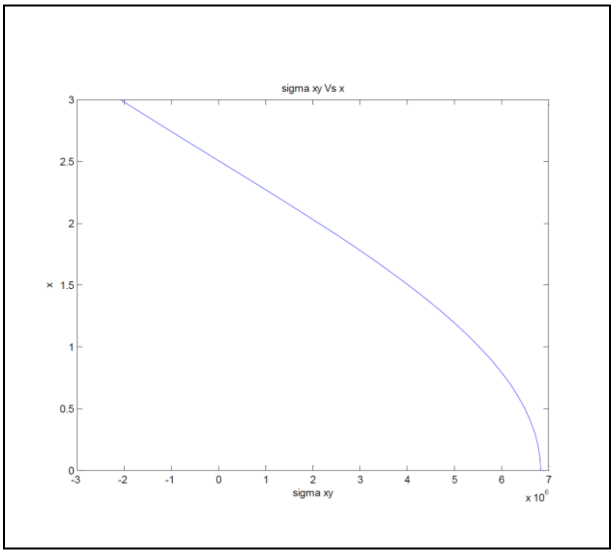

Figure 7: sigma xyVs x

From figure 7, initially when x=0 thermal stresses having maximum value chaning with value of x thermal stresses decreases.

VII.CONCLUSION

In this paper the problem is defined for the homogeneous boundary conditions of first, second and third kind. Temperature has been determined using finite Fourier sine transform finite Fourier cosine transform, finite MarchiFasulo transform,with internal moving heat source. The results are obtained in the form of infinite series. Also thermal stresses are determined numerically and graphically.

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