International Journal of Advanced Technology in Engineering and Science
Vol. No.4, Issue No. 03, March 2016
www.ijates.com
ijates
ISSN 2348 - 7550

SEMI COMPATIBILITY AND COMMON FIXED POINT IN INTUITIONISTIC FUZZY METRIC SPACE

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ABSTRACT

The purpose of this paper is to prove a common fixed point theorem for four maps by using the concept of semi compatible mappings in intuitionistic fuzzy metric space which is generalization of Singh et. al[11], Vasuki[13],Singh and Chouhan [10].

Keywords. Common fixed points, Intuitionistic fuzzy metric space, Compatible maps, Semi compatible mapping.

Mathematics Subject Classification. Primary 47H10, Secondary 54H25.

I. INTRODUCTION

It proved a turning point in the development of mathematics when the notion of fuzzy set was introduced by Zadeh[15], which laid the foundation of fuzzy mathematics. George and Veeramani [4] modified the concept of fuzzy metric space introduced by Kramosil and Michalek[6].They also showed that every metric space induces by fuzzy metric. In 1995 Cho, Sharma and Sahu[3] introduced the concept of semi compatibility of maps in d – complete metric spaces.

Atanassov[2] introduced and studied the concept of intuitionistic fuzzy set, Alaca et.al[1] defined the notion of intuitionistic fuzzy metric space, as Park[8] with the help of continuous t – norm and continuous t – conorm, as a generalization of fuzzy metric space. Turkoglu et. al[13] introduced the notion of Cauchy sequence in intuitionistic fuzzy metric space. They generalized the Jungck's [5] common fixed point theorem in intuitionistic fuzzy metric space and proved the intuitionistic fuzzy version of Pant's theorem [7] by giving the definition of weakly commuting and R-weakly commuting mapping in intuitionistic fuzzy metric space. Recently, Park [9] proved some common fixed point theorems in intuitionistic fuzzy metric spaces.

II. BASIC DEFINITIONS AND PRELIMINARIES.

We begin by briefly recalling some definitions and notations from fixed point theory literature that we will use in sequel, the concept of triangular norms (t - norm) and triangular conforms (t- conorms) were originally introduced by Schweizer and Skalar[10].

Definition 2.1. [10]. A binary operation $*:[0,1]\text{x}[0,1]\rightarrow[0,1]$ is called a *t-norm* * satisfies the following conditions:

i. * is commutative and associative,

ii. * is continuous,

iii. a * 1 =a for all a ∈ [0, 1],

iv. a * b ≤ c * d whenever a ≤ c and b ≤ d for all a, b, c, d ∈ [0,1].

Examples of *t-norm:* a * b = ab and a * b=min{a, b}.

Definition 2.2[1]. A binary operation ◊:[0,1]x[0,1]→[0,1] is said to be continuous t-co norm if it satisfied the following conditions:

i. ◊ is associative and commutative,

ii. ◊ is continuous,

iii. a ◊ 0 = a for all a ∈ [0,1],

iv. a ◊ b ≤ c ◊ d whenever a ≤ c and b ≤ d for each a, b, c, d ∈ [0,1]

Examples of t-conorm: a ◊ b = min(a+b, 1) and a ◊ b = max(a, b)

Remark 2.1.[10] The concept of triangular norms (*t-norm) and* triangular conorms (*t-conorm)* are knows as axiomatic skeletons that we use for characterizing fuzzy intersections and union respectively.

Definition 2.3. [1] A 5- tuple (X, M, N, *, ◊) is called intuitionistic fuzzy metric space if X is an arbitrary non empty set, * is a continuous *t-norm,* ◊ continuous *t-conorm* and M, N are fuzzy sets on X^2 x [0,∞] satisfying the following conditions:For each x, y, z, ∈X and t, s > 0

(IFM-1) M(x , y, t) + N(x, y, t) ≤ 1,

(IFM-2) M(x, y, 0) = 0, for all x, y in X,

(IFM-3) M(x, y, t) = 1 for all x, y in X and t > 0 if and only if x=y,

(IFM-4) $M(x, y, t) = M(y, x, t)$, for all x, y in X and t > 0,

(IFM-5) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$,

(IFM-6) $M(x, y, .): [0, \infty] \to [0,1]$ is left continuous,

(IFM-7) $\lim_{t\to\infty} M(x, y, t) = 1$,

(IFM-8) $N(x, y, 0) = 1$, for all x, y in X,

(IFM-9) $N(x, y, t) = 0$, for all x, y in X and t > 0 if and only if x = y,

(IFM-10) $N(x, y, t) = N(y, x, t)$, for all x, y in X and t>0,

(IFM-11) $N(x, y, t) \lozenge N(y, z, s) \geq N(x, z, t + s)$,

(IFM-12) $N(x, y, .): [0, \infty) \to [0,1]$ is right continuous,

(IFM-13) $\lim_{t\to\infty} N(x, y, t) = 0$, for all x, y in X and t > 0.

Then (M, N) is called an intuitionistic fuzzy metric on X. The function M(x, y, t) and N(x, y, t) denote the degree of nearness and degree of non- nearness between x and y with respect to t, respectively.

Remark 2.2.[13]. An Intuitionistic Fuzzy Metric space with continuous t-norm* and continuous *t-conorm* ◊ defined by a * a ≥ a, and (1-a) ◊ (1-a) ≤ (1-a)for all a∈ [0,1]. Then for all x, y∈ X, M(x, y, *) is non decreasing and N(x, y, ◊) is non increasing.

Example 2.1. [8] Let (X, d) be a metric space. Define a * b = ab and a ◊ b = min{1, a+b }, for all a, b ∈ [0, 1] and let M and N be fuzzy sets on X^2 x (0, ∞) defined as follows:

M(x, y, t) = $\frac{t}{t+d(x,y)}$ and N(x, y, t) = $\frac{d(x,y)}{t+d(x,y)}$ for all x, y ∈ X and all t > 0.

then (M, N) is called an intuitionistic fuzzy metric space on X. We call this intuitionistic fuzzy metric induced by a metric d the standard intuitionistic fuzzy metric.

Remark 2.3. Note that the above examples holds even with the t- norm a * b = min{a, b} and t- conorm a ◊ b= max{a, b}and hence (M, N) is an intuitionistic fuzzy metric with respect to any continuous t – norm and continuous t – conorm.

Lemma 2.1.[13] Let (X, M, N, *, ◊) Intuitionistic fuzzy metric space, If there exists k ∈ (0, 1) such that for all x, y ∈ X, M(x, y, kt) ≥ M(x, y, t) and , N(x, y, kt) ≤ N(x, y, t)
for all t > 0, then x = y.

Definition 2.4[1].A sequence $\{x_n\}$ in intuitionistic fuzzy metric space (X, M, N, *, ◊) is said to be:

(i) Cauchy sequence if for each t > 0,p > 0,

$$\lim_{n\to\infty} M(x_{n+p}, x_n, t) = 1 \text{ and } \lim_{n\to\infty} N(x_{n+p}, x_n, t) = 0.$$

(ii) Convergent to a point x ∈ X if for all t > 0,

$\lim_{n\to\infty} M(x_n, x, t) = 1$ and $\lim_{n\to\infty} N(x_n, x, t) = 0.$

An Intuitionistic Fuzzy metric space (X, M, N, *, ◊) is said to be complete if every Cauchy sequence in it converges to a point in it.

Definition 2.5.[1] A pair of self mapping (A, S) of a intuitionistic fuzzy metric space (X, M, N, *, ◊) is said to be compatible if $\lim_{n\to\infty} M(ASx_n, SAx_n, t) = 1 \; and \; \lim_{n\to\infty} N(ASx_n, SAx_n, t) = 0 \; for \; all \; t > 0$. Whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = u \; for \; some \; u \in X$.

Lemma 2.2.[2] Let $\{x_n\}$ be a sequence in Intuitionistic fuzzy metric space (X, M, N, *, ◊). If there exists a number k ∈ (0, 1) such that $M(x_{n+2}, x_{n+1}, kt) \geq M(x_{n+1}, x_n, t)$ and $N(x_{n+2}, x_{n+1}, kt) \leq N(x_{n+1}, x_n, t)$ for all t > 0, and n ∈ N. Then $\{x_n\}$ is Cauchy sequence in X.

Throughout in this section, let(X, M, *) denote a fuzzy metric space. We recall the following definition and propositions in fuzzy metric spaces.

Definition 2.6.[12] A pair (A, S) of self mappings of a fuzzy metric space is said to be semi compatible if $\lim_{n\to\infty} ASx_n = Sx$ whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = x$.

It follows that (A, S) is semi compatible and $Ay = Sy \; imply \; ASy = SAy$ by taking $\{x_n\}$ = y and $x = Ay = Sy$.

Definition 2.7. [9] A pair (A, S) of self mappings of an Intuitionistic fuzzy metric space is said to be semi compatible if $\lim_{n\to\infty} ASx_n = Sx$ whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = x$.

It follows that (A, S) is semi compatible and $Ay = Sy \; imply \; ASy = SAy$ by taking $\{x_n\}$ = y and $x = Ay = Sy$.

Proposition2.1. In intuitionistic fuzzy metric space (X, M, N,*,◊) limit of a sequence is unique.

For the proof of the above propositions\ one can use same techniques as in Singh et. al[12] .

Lemma 2.3.[9] Let A , B be self mappings of an intuitionistic fuzzy metric space (X, M, N,*,◊). If B is continuous then (A, B) is semi – compatible if and only if (A, B) is compatible.

III. MAIN RESULT.

In this section, a fixed point theorem for four self maps using the concept of semi compatible maps has been established which generalizes the results of Singh et. al [12], Park[9] from fuzzy metric space to intuitionistic

International Journal of Advanced Technology in Engineering and Science
Vol. No.4, Issue No. 03, March 2016
www.ijates.com
ijates
ISSN 2348 - 7550

fuzzy metric space. This result extends and generalizes many fixed point results in intuitionistic fuzzy metric space.

Theorem 3.1. Let A, B, S and T be self mappings of a complete intuitionistic fuzzy metric space (X, M, N, *, ◊) satisfying

I. $A(X) \subset T(X), B(X) \subset S(X)$

II. One of A, B, S and T is continuous.

III. Pairs (A, S) and (B, T) are semi compatible.

IV. $\exists$ *some* $k \in (0,1)$ such that for all $x, y \in X, t > 0$.

$$M(Ax, By, kt) \geq \text{Min}\{M(Sx, Ty, t), M(Sx, Ax, t), M(Ty, By, t), M(Sx, By, 2t), M(Ty, Ax, t)\}$$

$$N(Ax, By, kt) \leq \text{Max}\{N(Sx, Ty, t), N(Sx, Ax, t), N(Ty, By, t), N(Sx, By, 2t), N(Ty, Ax, t)\}$$

V. for all $x, y \in X, M(x, y, t) \to 1$ and $N(x, y, t) \to 0$ as $t \to \infty$.

Then A, B, S and T have a unique common fixed point.

Proof. Let $x_0 \in X$ be any point. Since $A(X) \subset T(X), B(X) \subset S(X)$, $\exists\, x_1, x_2 \in X$ such that $Ax_0 = Tx_1$ *and* $Bx_1 = Sx_2$. Inductively we construct a sequence $\{y_n\}$ in X such that $y_{2n+1} = Ax_{2n} = Tx_{2n+1}$ and $y_{2n+2} = Bx_{2n+1} = Sx_{2n+2}$,($y_{2n} = Sx_{2n}$), n = 0, 1, 2, …..

Using IV, we have

$M(y_{2n+1}, y_{2n+2}, kt) = M(Ax_{2n}, Bx_{2n+1}, kt)$

$\geq Min\{M(Sx_{2n}, Tx_{2n+1}, t), M(Sx_{2n}, Ax_{2n+1}, t), M(Tx_{2n+1}, Bx_{2n+1}, t), \quad M(Sx_{2n}, Bx_{2n+1}, 2t), M(Tx_{2n+1}, Ax_{2n}, t)\}$

$\geq Min\,\{M(y_{2n}, y_{2n+1}, t), M(y_{2n}, y_{2n+1}, t), M(y_{2n+1}, y_{2n+2}, t), M(y_{2n}, y_{2n+2}, 2t), M(y_{2n+1}, y_{2n+1}, t)\}$

$\geq Min\,\{M(y_{2n}, y_{2n+1}, t), M(y_{2n+1}, y_{2n+2}, t), M(y_{2n}, y_{2n+1}, t), M(y_{2n+1}, y_{2n+2}, 2t), 1\}$

$$\geq Min\,\{M(y_{2n}, y_{2n+1}, t), M(y_{2n+1}, y_{2n+2}, t)\} = M(y_{2n+1}, y_{2n}, t)$$

and

$N(y_{2n+1}, y_{2n+2}, kt) = N(Ax_{2n}, Bx_{2n+1}, kt)$

$\leq Max\{N(Sx_{2n}, Tx_{2n+1}, t), N(Sx_{2n}, Ax_{2n+1}, t), N(Tx_{2n+1}, Bx_{2n+1}, t), \quad N(Sx_{2n}, Bx_{2n+1}, 2t), N(Tx_{2n+1}, Ax_{2n}, t)\}$

$\leq Max\{N(y_{2n}, y_{2n+1}, t), N(y_{2n}, y_{2n+1}, t), N(y_{2n+1}, y_{2n+2}, t), N(y_{2n}, y_{2n+2}, 2t), N(y_{2n+1}, y_{2n+1}, t)\}$

$\leq Max\{N(y_{2n}, y_{2n+1}, t), N(y_{2n+1}, y_{2n+2}, t), N(y_{2n}, y_{2n+1}, t), N(y_{2n+1}, y_{2n+2}, 2t), 0\}$

$\leq Max\{N(y_{2n}, y_{2n+1}, t), N(y_{2n+1}, y_{2n+2}, t)\} = N(y_{2n+1}, y_{2n}, t)$.

Since $M(x, y, t)$ is non – decreasing, and $N(x, y, t)$ is non – increasing.

Similarly we have $\quad M(y_{2n+1}, y_{2n}, kt) \geq M(y_{2n}, y_{2n-1}, t)$

and $\quad N(y_{2n+1}, y_{2n}, kt) \leq N(y_{2n}, y_{2n-1}, t)$.

Hence $\quad M(y_{n+1}, y_n, kt) \geq M(y_n, y_{n-1}, t)$,

and $\quad N(y_{n+1}, y_n, kt) \leq N(y_n, y_{n-1}, t)$, for all n.

We show that $\lim_{n\to\infty} M(y_{n+p}, y_n, kt) = 1$ *and* $\lim_{n\to\infty} N(y_{n+p}, y_n, kt) = 0$ for all p and t > 0.

Now $\quad M(y_{n+1}, y_n, kt) \geq M\left(y_n, y_{n-1}, {}^{t}/_{k}\right)$

$$\geq M\left(y_n, y_{n-1}, {}^{t}/_{k^2}\right)$$

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ISSN 2348 - 7550

$$> M\left(y_1, y_0, {}^{t}/_{k^n}\right) \to 1 \; as \; {}^{t}/_{k^n} \to \infty \; as \; n \to \infty$$

and $N(y_{n+1}, y_n, kt) \leq N\left(y_n, y_{n-1}, {}^{t}/_{k}\right)$

$$\leq N\left(y_n, y_{n-1}, {}^{t}/_{k^2}\right)$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$< N\left(y_1, y_0, {}^{t}/_{k^n}\right) \to 0 \; as \; {}^{t}/_{k^n} \to \infty \; as \; n \to \infty .$$

Thus the result holds for p = 1. By induction hypothesis suppose that the result holds for p = r. Now

M$(y_n, y_{n+r+1}, t) \geq M\left(y_n, y_{n+r}, {}^{t}/_{2}\right) * M\left(y_{n+r}, y_{n+r+1}, {}^{t}/_{2}\right) \to 1 * 1 = 1$

and N$(y_n, y_{n+r+1}, t) \leq N\left(y_n, y_{n+r}, {}^{t}/_{2}\right) * N\left(y_{n+r}, y_{n+r+1}, {}^{t}/_{2}\right) \to 0 * 0 = 0$.

Thus the result holds for $p = r + 1$.

Hence $\{y_n\}$ is a Cauchy sequence in X and as X is complete we get $\{y_n\} \to z \in X$.

Therefore $Ax_{2n} \to z, \quad Sx_{2n} \to z$ (3.1)

$Tx_{2n+1} \to z, \quad Bx_{2n+1} \to z.$ (3.2)

Case I. S is continuous.

In this case $SAx_{2n} \to Sz,$ (3.3)

$S^2x_{2n} \to Sz$ (3.4)

Also (A, S) is semi compatible

$ASx_{2n} \to Sz,$ (3.5)

Step 1. Take $x = Sx_{2n}, y = x_{2n+1}$ in IV we get

M$(ASx_{2n}, Bx_{2n+1}, kt) \geq Min\{M(SSx_{2n}, Tx_{2n+1}, t), M(SSx_{2n}, \text{A}Sx_{2n+1}, t), M(Tx_{2n+1}, Bx_{2n+1}, t), M(SSx_{2n},$
$Bx_{2n+1}, 2t), M(Tx_{2n+1}, ASx_{2n}, t)\}$

and N$(ASx_{2n}, Bx_{2n+1}, kt) \leq Max\{N(SSx_{2n}, Tx_{2n+1}, t), N(SSx_{2n},$
$\text{A}Sx_{2n+1}, t), N(Tx_{2n+1}, Bx_{2n+1}, t), N(SSx_{2n}, Bx_{2n+1}, 2t), N(Tx_{2n+1}, ASx_{2n}, t)\}$

Taking limit $n \to \infty$ and using equation (3.1) to (3.5) we get

$$M(Sz, z, kt) \geq Min\{M(Sz, z, t), M(Sz, Sz, t), M(z, z, t), M(Sz, z, 2t), M(z, Sz, t)\}$$

$$\geq Min\{M(Sz, z, t), M(Sz, z, 2t)\} = M(Sz, z, t)$$

and $N(Sz, z, kt) \leq Max\{N(Sz, z, t), N(Sz, Sz, t), N(z, z, t), N(Sz, z, 2t), N(z, Sz, t)\}$

$$\leq Max\{N(Sz, z, t), N(Sz, z, 2t)\} = N(Sz, z, t)$$

Hence $Sz = z.$ (3.6)

Step 2. Put $x = z, y = x_{2n+1}$ in IV we have

M$(Az, Bx_{2n+1}, kt) \geq Min\{M(Sz, Tx_{2n+1}, t), M(Sz, \text{A}z, t), M(Tx_{2n+1}, Bx_{2n+1}, t), M(Sz,$
$Bx_{2n+1}, 2t), M(Tx_{2n+1}, Az, t)\}$

and N$(Az, Bx_{2n+1}, kt) \leq Max\{N(Sz, Tx_{2n+1}, t), N(Sz, \text{A}z, t), N(Tx_{2n+1}, Bx_{2n+1}, t), N(Sz,$
$Bx_{2n+1}, 2t), N(Tx_{2n+1}, Az, t)\}$

Taking limit $n \to \infty$ and using equation (3.2) we get

$$M(Az, z, kt) \geq Min\{M(Sz, z, t), M(z, Az, t), M(z, z, t), M(Sz, z, 2t), M(z, Az, t)\}$$

$$= M(z, Az, t)$$

and

$N(Az, z, kt) \leq Max\{N(Sz, z, t), N(z, Az, t), N(z, z, t), N(Sz, z, 2t), N(z, Az, t)\}$

$= N(z, Az, t)$, using (3.6)

Thus $Az = z = Sz.$ (3.7)

Step 3. $A(X) \subset T(X)$, $\exists\ u \in X$ such that

$Az = Tu = z$ (3.8)

Put $x = x_{2n}$, y = u in IV we have

M(Ax_{2n}, Bu, kt)≥ $Min\{M(Sx_{2n}, Tu, t), M(Sx_{2n}, Ax_{2n}, t), M(Tu, Bu, t), M(Sx_{2n}, Bu, 2t), M(Tu, Ax_{2n}, t)\}$

N(Ax_{2n}, Bu, kt)≤ $Max\{N(Sx_{2n}, Tu, t), N(Sx_{2n}, Ax_{2n}, t), N(Tu, Bu, t), N(Sx_{2n}, Bu, 2t), N(Tu, Ax_{2n}, t)\}$

Taking limit $n \to \infty$ and using equation (3.1) we get

$M(z, Bu, kt) \geq Min\{M(z, Tu, t), M(z, z, t), M(Tu, Bu, t), M(z, Bu, 2t), M(Tu, z, t)\}$

$\geq Min\{M(z, z, t), 1, M(z, Bu, t), M(z, Bu, 2t), M(z, z, t)\}$

$\geq Min\{M(z, Bu, 2t), M(z, Bu, t)\} = M(z, Bu, t)$

and $N(z, Bu, kt) \leq Max\{N(z, Tu, t), N(z, z, t), N(Tu, Bu, t), N(z, Bu, 2t), N(Tu, z, t)\}$

$\leq Max\{N(z, z, t), 0, N(z, Bu, t), N(z, Bu, 2t), N(z, z, t)\}$, using (3.8)

$\leq Max\{N(z, Bu, 2t), N(z, Bu, t)\} = N(z, Bu, t)$.

Thus $z = Bu$ and we get $Tu = Bu = z$, and since (B, T) is semi compatible we get $BTu = TBu\ i.e. Bz = Tz$.

Step 4. Take x = z y = z in IV

$M(Az, Bz, kt) \geq Min\{M(Sz, Tz, t), M(Sz, Az, t), M(Tz, Bz, t), M(Sz, Bz, 2t), M(Tz, Az, t)\}$,

and $N(Az, Bz, kt) \leq Max\{N(Sz, Tz, t), N(Sz, Az, t), N(Tz, Bz, t), N(Sz, Bz, 2t), N(Tz, Az, t)\}$,

$As\quad Az = Sz = z\ and\ Bz = Tz$ we have

$M(z, Tz, kt) \geq Min\{M(z, Tz, t), M(z, z, t), M(Tz, Tz, t), M(z, Tz, 2t), M(Tz, z, t)\}$,

$M(z, Tz, kt) \geq Min\{M(z, Tz, t), M(z, Tz, 2t)\} = M(z, Tz, t)$

and

$N(z, Tz, kt) \leq Max\{N(z, Tz, t), N(z, z, t), N(Tz, Tz, t), N(z, Tz, 2t), N(Tz, z, t)\}$,

$N(z, Tz, kt) \leq Max\{N(z, Tz, t), N(z, Tz, 2t)\} = N(z, Tz, t)$.

Thus $z = Tz$. Therefore $Az = Sz = Bz = Tz = z$.

Hence z is common fixed point of $A, S, B\ and\ T$.

Uniqueness. Let z and z' be two common fixed points of the maps $A, S, B\ and\ T$. Then $Az = Sz = Bz = Tz = z$ and $z' = Sz' = Bz' = Tz' = z'$. Using IV, we get

$M(z, z', kt) = M(Az, Bz', kt) \geq$

$Min\{M(Sz, Tz', t), M(Sz, Az, t), M(Tz', Bz', t), M(Sz, Bz', 2t), M(Tz', Az, t)\}$,

$\geq Min\{M(z, z', t), M(z, z, t), M(z', z', t), M(z, z', 2t), M(z, z', t)\}$

$\geq Min\{M(z, z', t), M(z, z', 2t)\} = M(z, z', t)$

and $N(z, z', kt) = N(Az, Bz', kt)$

$\leq Max\{N(Sz, Tz', t), N(Sz, Az, t), N(Tz', Bz', t), N(Sz, Bz', 2t), N(Tz', Az, t)\}$,

$\leq Max\{N(z, z', t), N(z, z, t), N(z', z', t), N(z, z', 2t), N(z, z', t)\}$

$\leq Max\{N(z, z', t), N(z, z', 2t)\} = N(z, z', t)$.

International Journal of Advanced Technology in Engineering and Science
Vol. No.4, Issue No. 03, March 2016
www.ijates.com
ijates
ISSN 2348 - 7550

Thus $z = z'$. Hence z is the unique common fixed point of the four maps A, B, S and T.

Case 2. A is continuous.

Using (3.1) and (3.2) and continuity of A we get

$A^2 z_{2n} \rightarrow Az$ (3.9)

$ASx_{2n} \rightarrow Az$ (3.10)

Since (A, S) is semi compatible

$\lim_{n\rightarrow\infty} ASx_{2n} \rightarrow Sz$ (3.11)

Using (3.10) and (3.11) we get $Az = Sz$, since limit s unique.

And rest of the proof follows from step II on ward previous case.

Similarly we can prove the result using the continuity of B or T.

3.1 Corollary

Let A, B, S and T be four self maps of a complete intuitionistic fuzzy metric space (X, M, N, *, ◊) with continuous t – norm * and continuous t – conorm ◊ defined by a * b=min{a, b}and a ◊ b = max(a, b) satisfying (I), (II), (III), (V) and for all$x, y \in X, t > 0 \; \exists \; some \; k \in (0,1)$ such that

$(IV')M(Ax, By, kt) > M(Sx, Ty, t)$.

Then A, B, S and T have a unique common fixed point.

Proof. Here we have only one factor in condition (IV') as four factors as in condition (IV) of Theorem 3.1, thus the proof can be given on the line of that of Theorem 3.1.

3.2 Corollary

Let A, B, S and T be four maps of a complete intuitionistic fuzzy metric space (X, M, N, *, ◊) with continuous t – norm * and continuous t – conorm ◊ defined by a * b=min{a, b}and a ◊ b = max(a, b) satisfying

I. $A^m(X) \subset T^p(X), B^n(X) \subset S^q(X)$, where n, m, p, q $\in N$

II. $AS = SA, TB = BT$

III. S and T are continuous.

IV. Pairs (A^m, S^q) and (B^n, T^p) are semi compatible.

V. for all $x, y \in X, M(x, y, t) \rightarrow 1$ and $N(x, y, t) \rightarrow 0$ as $t \rightarrow \infty$.

VI. $\exists \; some \; k \in (0,1)$ such that for all $x, y \in X, t > 0$.

$M(A^m x, B^n y, kt)$

$\geq \text{Min}\{M(S^q x, T^p y, t), M(S^q x, A^m x, t), M(T^p y, B^n y, t), M(S^q x, B^n y, 2t), M(T^p y, A^m x, t)\}$

and

$N(A^m x, B^n y, kt) \leq \text{Max}\{N(S^q x, T^p y, t), N(S^q x, A^m x, t), N(T^p y, B^n y, t), N(S^q x, B^n y, 2t), N(T^p y, A^m x, t)\}$

Then A, B, S and T have a unique common fixed point.

Proof. Since $AS = SA \; and \; TB = BT$ we get $A^m S^q = S^q A^m$ and $B^n T^p = T^p B^n$. Since S and T are continuous we get $S^q \; and \; T^p$ are also continuous. We want to show (A^m, S^q) and (B^n, T^p) are semi compatible. Consider the sequences $\{A^m x_i\}$ and $\{S^q x_i\}$ which converge to x. As S^q is continuous we get $\{S^q A^m x_i\}$ converges to $S^q x$ which imply $\{A^m S^q x_i\}$converge to $S^q x$.

Thus (A^m, S^q) is semi compatible. Similarly (B^n, T^p) is semi compatible. By theorem 3.1, A^m, B^n, S^q and T^p have a unique common fixed point z i.e. $A^m z = B^n z = S^q z = T^p z = z$.

Now $Az = A(A^m z) = A^m(Az) and\ Az = A(S^q z) = S^q Az$. Hence Az is a common fixed point of $A^m\ and\ S^q$.

Similarly Bz is common fixed point of B^n and T^p.

Now put $x = Az\ and\ y = Bz$ in IV, we get $= Bz$.

Hence $z = Az = Bz$. Similarly we can prove $z = Sz = Tz$.

Thus we have $z = Az = Bz = Sz = Tz$.

Hence z is the unique common fixed point of $A, B, S\ and\ T$.

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