International Journal of Advanced Technology in Engineering and Science
Vol. No.3, Issue 11, November 2015
www.ijates.com
ijates
ISSN 2348 - 7550

STUDIES ON STABILITY BEHAVIOUR OF NANOBARS USING NONLOCAL ELASTICITY THEORY

Princy Babu John[1], M.G Rajendran[2]

[1]P G Student, [2]Professor, School of Civil Engineering,

Karunya University Coimbatore,Tamil Nadu, (India)

ABSTRACT

This paper investigates the analysis for buckling response of nanobar with various end conditions using Eringen's nonlocal elasticity theory. The governing equations for the buckling of nanobar is formulated using Euler- Bernoulli Beam theory to study the effect of the small- scale parameter on the buckling behaviour of nanobars. The small- scale parameter is taken into consideration by using Eringen's nonlocal elasticity theory. The analytical solutions are obtained for simply supported, clamped- clamped, clamped- hinged and cantilever end conditions. The effects of the nonlocal parameter on the buckling loads are studied. The results and the available solutions are compared and the buckling loads for all boundary conditions are found to be in excellent agreement with existing results.

Keywords: Buckling Loads, Boundary Conditions, Eringen's Nonlocal Elasticity, Nonlocal Parameter.

I. INTRODUCTION

Carbon nanotubes were discovered by Iijima [1] in 1991.Vibration and buckling problems of straight carbon nanotubes (CNT) occupy an important place in micro- and nano-scale devices and systems. Examples include nanosensors, nanoactuators, nanooscillators, micro-resonators and field emission devices, etc. In order to make full potential application of CNT, it is essential to understand their mechanical behavior well. In many papers, analytical analyses of the mechanical behavior of CNT have been proposed besides the experimental work by Carbon nanotubes can be modeled using atomistic or continuum mechanics methods. The atomic methods are limited to systems with a small number of molecules or atoms and therefore they are restricted to the study of small scale modeling. Unlike atomistic modeling, continuum models view CNT as a continuous beam.

C. M. Wang et al. [2] reviews recent research studies on the buckling of carbon nanotubes. The structure and properties of carbon nanotubes are introduced. The various buckling behaviours exhibited by carbon nanotubes are also presented. It also found that CNTs have the remarkable flexibility and stability under external loading. Metin Aydogdu [7] developed the Nonlocal elastic rod model and applied it to investigate the small scale effect on axial vibration of the nanorods. In this study generalized non local beam theory is proposed to study bending, buckling free vibrations of nanobars. Nonlocal constitutive equations of Eringen are used in the formulations. Q.Wang, K.M.Liew [5] investigated the local buckling of carbon nanotubes under bending. Devesh Kumar et al.

[10] studied the buckling of carbon nanotubes, modelled as nonlocal one dimensional continuum within the framework of Euler–Bernoulli beams. The authors of this paper are of the opinion that the studies here reported are not sufficient to thoroughly understand the effect of small scale parameter and the limitations of classical elastic theory in dealing with the length scale of nanobars.

For realistic analysis of CNT, one must incorporate small-scale effects to achieve solutions with acceptable accuracy. Since the classical continuum models are scale free, for the modeling of CNT structures one can use modified elasticity theories like Eringen theory (Eringen, 1983; Eringen and Edelen, 1972) or strain gradient theories (Lam *et al.*, 2003; Kong *et al.*, 2009; Akg¨oz and Civalek, 2011). In this way, the internal size scale could be considered in the constitutive equation simply as a material parameter. In the theory of nonlocal elasticity, the stress at a reference point is considered to be a functional of the strain field at every point in the body. It can be concluded that continuum mechanics with size-effect could potentially play a useful role in the analysis related to nanostructures. The first application of the Eringen nonlocal constitutive relation on the Euler-Bernoulli beam is the work of Peddinson *et al*. (2003).

1.1 Nonlocal Elasticity Theory

In the analysis of macro beams, the classical theory of elasticity is used. But when the small scale/ nano scale (e.g. CNT) is taken into account, the classical theory does not hold good. Hence the Nonlocal elasticity which was proposed by Eringen is adopted to account for small scale effect in elasticity by assuming the stress at a reference point to be a functional of the strain field at every point in the body. In this way, the internal size scale could be considered in the constitutive equations simply as a material parameter. The application of nonlocal elasticity, in micro and nanomaterials, has received much attention among the nanotechnology community recently. This important length scale effect is used in vibration, buckling and bending of CNTs studies. The application of nonlocal elasticity is recommend in revealing scale effects for nano-materials like CNTs.

According to A.Cemal Eringen [3], the nonlocal stress tensor σ at point X is expressed as:

$$\sigma = \int_V K(|\mathbf{x}' - \mathbf{x}|, \tau)\mathbf{t}(\mathbf{x}')d\mathbf{x}',$$

Use of integral constitutive relations is relatively more difficult in computation than using algebraic or differential constitutive relations. Realizing this fact Eringen proposed an equivalent differential model as:

$$(1 - {\mu_0}^2\nabla^2)\bar{\sigma} = \sigma, \mu_0 = \tau^2.l^2 = {e_0}^2.a^2$$

σ_{xx}, ε_{xx} and E are a normal stress, a normal strain and Young's modulus, respectively.

$\mu = {e_0}^2 a^2$ is the function of material constant,

$e_0 =$ material constant

a = internal characteristic lengths (such as the lattice spacing).

$e_0 a$ is called the nonlocal parameter which is a factor to consider the effect of small length scale.

II. FORMULATION

2.1 Formulation of Governing Equation

The governing equation for the stability analysis of a nanobar is based on the assumptions of Euler- Bernoulli beam theory and is derived as follows.

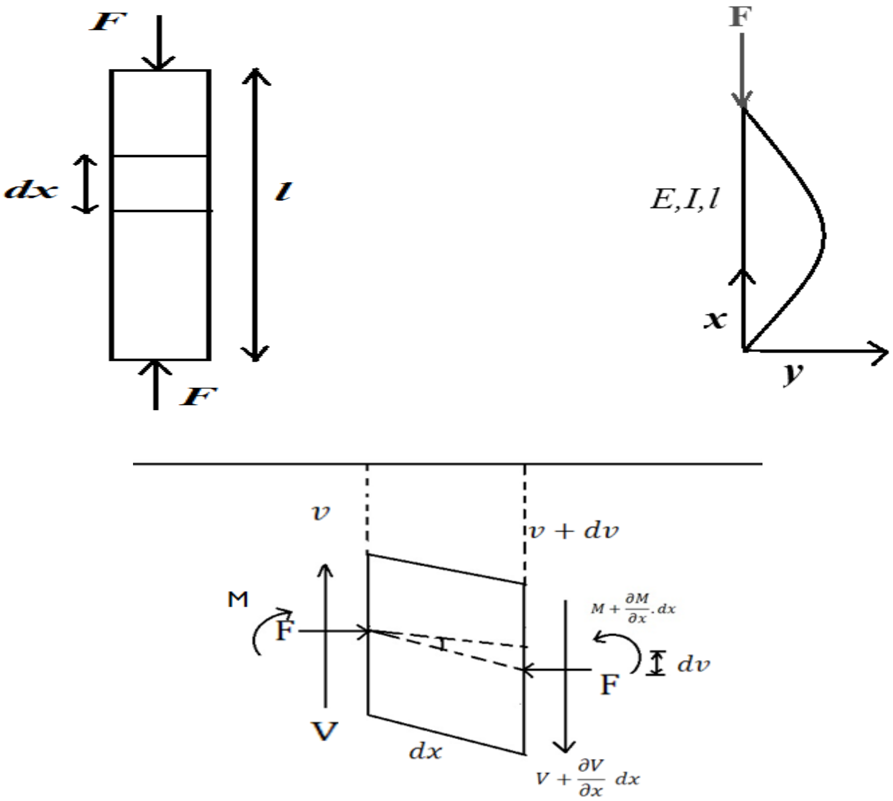


Fig.1 Equilibrium Condition of a Differential Element of a Nanobar Subjected to Axial Loads.

Force equilibrium in vertical direction,

$\sum Fy = 0$

$$\frac{dV}{dx} = 0 \tag{1}$$

Taking moment about o,

$\sum M = 0$

$$V = \frac{dM}{dx} - F\left(\frac{dv}{dx}\right) \tag{2}$$

Substitute Eqn.(2) into Eqn.(1),

$$\frac{d^2M}{dx^2} - F\left(\frac{d^2v}{dx^2}\right) = 0 \tag{3}$$

We know,

$$M = \int \sigma \, dA.\, y \tag{4a}$$

$$\varepsilon x = \frac{\partial u}{\partial x} = -y\left(\frac{d^2v}{dx^2}\right) \tag{4b}$$

Using Eringen's Nonlocal Constitutive Law,

$$\sigma x - (e_0 a)^2 \frac{d^2}{dx^2}(\sigma x) = E\varepsilon(x)$$

$$\sigma x = (e_0 a)^2 \frac{d^2}{dx^2}(\sigma x) + E\varepsilon(x) \tag{5}$$

Substitute (Eqn.5) into Eqn.(4a),

$$M = \iint [(e_0 a)^2 \frac{d^2 \sigma(x)}{dx^2} - E\varepsilon(x)]\, y.dA$$

Making use of Eqn. ④b,

$$M = \iint [(e_0 a)^2 \frac{d^2}{dx^2}\sigma(x) - Ey\,\frac{d^2 v}{dx^2}]\, y.dA$$

$$M = (e_0 a)^2 \frac{d^2}{dx^2}\int \sigma(x).y.dA - \int Ey^2 \frac{d^2 v}{dx^2} dA$$

$$M = (e_0 a)^2 \frac{d^2 M}{dx^2} - EI\frac{d^2 v}{dx^2}$$

$$M - (e_0 a)^2 \frac{d^2 M}{dx^2} = EI\frac{d^2 v}{dx^2} \tag{6}$$

From (Eqn.6),

$$\frac{d^2 M}{dx^2} = -\frac{1}{(e_0 a)^2}\left[-M - EI\frac{d^2 v}{dx^2}\right] \tag{6a}$$

Considering Eqns (3) and (2),

$$\frac{d^2 M}{dx^2} = F\frac{d^2 v}{dx^2} \tag{3}$$

Eqn.(3) = (Eqn.6a)

$$\frac{1}{(e_0 a)^2}\left[M + EI\,\frac{d^2 v}{dx^2}\right] = F\frac{d^2 v}{dx^2}$$

$$M = F(e_0 a)^2 \frac{d^2 v}{dx^2} - EI\frac{d^2 v}{dx^2} \tag{7}$$

Substitute Eqn.(7) into Eqn.(2),

$$V = \frac{dM}{dx} = -EI\,\frac{d^3 v}{dx^3} + F(e_0 a)^2 \frac{d^3 v}{dx^3} - F\,\frac{dv}{dx} \tag{8}$$

Substitute Eqn.(8) into Eqn.(1),

$$-EI\frac{d^4 v}{dx^4} + F(e_0 a)^2 \frac{d^4 v}{dx^4} - F\frac{d^2 v}{dx^2} = 0$$

$$\boldsymbol{EI\frac{d^4 v}{dx^4} - F(e_0 a)^2 \frac{d^4 v}{dx^4} + F\frac{d^2 v}{dx^2} = 0}$$

2.2 Column Buckling Equation Using Nonlocal Elasticity Theory

$$EI\frac{d^4 v}{dx^4} + F\frac{d^2 v}{dx^2} - F(e_0 a)^2 \frac{d^4 v}{dx^4} = 0$$

$$\frac{d^4 v}{dx^4}[EI - F(e_0 a)^2] + F\frac{d^2 v}{dx^2} = 0$$

$$\frac{d^4 v}{dx^4} + \left[\frac{F}{EI - F(e_0 a)^2}\right]\frac{d^2 v}{dx^2} = 0$$

Let $k^2 = \frac{F}{EI - F(e_0 a)^2}$

(or) $k = \sqrt{\frac{F}{EI - F(e_0 a)^2}}$

$$\frac{d^4 v}{dx^4} + k^2\frac{d^2 v}{dx^2} = 0$$

$$(D^4 + k^2 D^2)v = 0$$

$\lambda^4 + k^2\lambda^2 = 0$

$\lambda^2(\lambda^2 + k^2) = 0$

The roots are :

$\lambda = \pm 0 \quad \Rightarrow (A + Bx)$

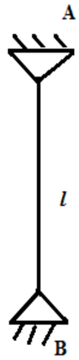
$\lambda = i \pm \beta \quad \Rightarrow e^0(C\ cos\beta x + D\ sin\beta x)$ (Here i=0, β=k)

The solution is:

$v(x) = A + Bx + C\cos kx + D\sin kx$

III. ANALYSIS OF NANO BAR WITH VARIOUS BOUNDARY CONDITIONS

3.1 Simply Supported end Conditions



$\boldsymbol{v(x) = A + Bx + C\ cos\ kx + D\ sin\ kx}$

$\boldsymbol{v''(x) = -Ck^2\ cos\ kx - Dk^2 sin\ kx}$

$\boldsymbol{v(l) = A + Bl + C\ cos\ kl + D\ sin\ kl}$

$\boldsymbol{v''(l) = -Ck^2\ cos\ kl - Dk^2 sin\ kl}$

The boundary conditions are:

(i) $v(0) = 0$

(ii) $v''(0) = 0$

(iii) $v(l) = 0$

(iv) $v''(l) = 0$

$$\begin{bmatrix} v(0) \\ v''(0) \\ v(l) \\ v''(l) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & -k^2 & 0 \\ 1 & l & \cos kl & \sin kl \\ 0 & 0 & -k^2\cos kl & -k^2\sin kl \end{bmatrix} \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & l & \cos kl & \sin kl \\ 0 & 0 & \cos kl & \sin kl \end{bmatrix} \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

It is an Eigen value problem.

$$\begin{vmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & l & \cos kl & \sin kl \\ 0 & 0 & \cos kl & \sin kl \end{vmatrix} = 0$$

$$1\begin{vmatrix} 0 & 1 & 0 \\ l & \cos kl & \sin kl \\ 0 & \cos kl & \sin kl \end{vmatrix} + 1\begin{vmatrix} 0 & 0 & 0 \\ 1 & l & \sin kl \\ 0 & 0 & \sin kl \end{vmatrix} = 0$$

$$l \sin kl = 0$$

$$\sin kl = 0$$

$$kl = n\pi$$

$$k = \frac{n\pi}{l}$$

$$k^2 = \frac{n^2\pi^2}{l^2}$$

$$\frac{F}{EI - F(e_0 a)^2} = \frac{n^2\pi^2}{l^2}$$

$$Fl^2 = n^2\pi^2 EI - F(e_0 a)^2 n^2\pi^2$$

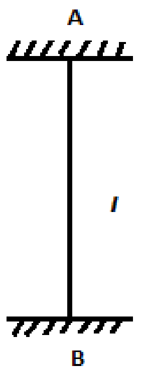
$$F\left[1 + (e_0 a)^2 \frac{n^2\pi^2}{l^2}\right] = \frac{n^2\pi^2 EI}{l^2}$$

$$F_{cr} = \frac{\frac{n^2\pi^2 EI}{l^2}}{1 + (e_0 a)^2 \frac{n^2\pi^2}{l^2}}$$

For smaller buckling loads, (n =1).The critical buckling load for column is given by,

$$F_{cr} = \frac{\frac{\pi^2 EI}{l^2}}{1 + (e_0 a)^2 \frac{\pi^2}{l^2}}$$

3.2 Clamped- Clamped end Conditions



$$v(x) = A + Bx + C\cos kx + D\sin kx$$

$$v'(x) = B - Ck\sin kx + Dk\cos kx$$

$$v(l) = A + Bl + C\cos kl + D\sin kl$$

$$v'(l) = B - Ck\sin kl + Dk\cos kl$$

The boundary conditions are:

(i) $v(0) = 0$

(ii) $v(l) = 0$

(iii) $v'(0) = 0$

(iv) $v'(l) = 0$

$$\begin{bmatrix} v(0) \\ v'(0) \\ v(l) \\ v'(l) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & k \\ 1 & l & \cos kl & \sin kl \\ 0 & 1 & -k\sin kl & k\cos kl \end{bmatrix} \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

It is an Eigen value problem.

$$\begin{vmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & k \\ 1 & l & \cos kl & \sin kl \\ 0 & 1 & -k\sin kl & k\cos kl \end{vmatrix} = 0$$

$$1\begin{vmatrix} 1 & 0 & k \\ l & \cos kl & \sin kl \\ 1 & -k\sin kl & k\cos kl \end{vmatrix} + 1\begin{vmatrix} 0 & 1 & k \\ 1 & l & \sin kl \\ 0 & 1 & k\cos kl \end{vmatrix} = 0$$

$$k - k^2 l \sin kl - 2k\cos kl + k = 0$$

$$-k^2 l \sin kl - 2k\cos kl + 2k = 0$$

$$kl\sin kl + 2(\cos kl - 1) = 0$$

$$2kl\, sin\frac{kl}{2}.\, cos\frac{kl}{2} + 2\left(1 - 2\, sin^2\frac{kl}{2} - 1\right) = 0$$

$$sin\frac{kl}{2}\left(\frac{kl}{2}.\, cos\frac{kl}{2} - 2\, sin\frac{kl}{2}\right) = 0$$

$$sin\frac{kl}{2} = 0$$

$$\frac{kl}{2} = n\pi$$

$$\frac{k^2 l^2}{4} = n^2\pi^2$$

$$k^2 = \frac{4n^2\pi^2}{l^2}$$

$$\frac{F}{EI - F(e_0 a)^2} = \frac{4n^2\pi^2}{l^2}$$

$$Fl^2 = 4n^2\pi^2 EI - 4F(e_0 a)^2 n^2\pi^2$$

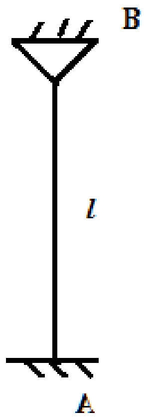
$$F\left[1 + \frac{4n^2\pi^2}{l^2}(e_0 a)^2\right] = \frac{4n^2\pi^2 EI}{l^2}$$

$$F_{cr} = \frac{\frac{4n^2\pi^2 EI}{l^2}}{1 + \frac{4n^2\pi^2}{l^2}(e_0 a)^2}$$

For smaller buckling loads, (n =1).The critical buckling load for column is given by,

$$\boldsymbol{F_{cr}} = \frac{\frac{4\pi^2 EI}{l^2}}{1 + \frac{4\pi^2}{l^2}(e_0 a)^2}$$

3.3 Clamped- Hinged end Conditions



$$v(x) = A + Bx + C\cos kx + D\sin kx$$

$$v'(x) = B - Ck\sin kx + Dk\cos kx$$

$$v''(x) = -Ck^2\cos kx - Dk^2\sin kx$$

$$v(l) = A + Bl + C\cos kl + D\sin kl$$

$$v'(l) = B - Ck\sin kl + Dk\cos kl$$

$$v''(l) = -Ck^2\cos kl - Dk^2\sin kl$$

The boundary conditions are:

(i) $v(0) \ = 0$

(ii) $v'(0) \ = 0$

(iii) $v(l) \ = 0$

(iv) $v'''(l) \ = 0$

$$\begin{bmatrix} v(0) \\ v'(0) \\ v(l) \\ v''(l) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & k \\ 1 & l & \cos kl & \sin kl \\ 0 & 0 & -k^2\cos kl & -k^2\sin kl \end{bmatrix} \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} v(0) \\ v'(0) \\ v(l) \\ v''(l) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & k \\ 1 & l & \cos kl & \sin kl \\ 0 & 0 & \cos kl & \sin kl \end{bmatrix} \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

It is an Eigen value problem.

$$\begin{vmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & k \\ 1 & l & \cos kl & \sin kl \\ 0 & 0 & \cos kl & \sin kl \end{vmatrix} = 0$$

$$1\begin{vmatrix} 1 & 0 & k \\ l & \cos kl & \sin kl \\ 0 & \cos kl & \sin kl \end{vmatrix} + 1\begin{vmatrix} 0 & 1 & k \\ 1 & l & \sin kl \\ 0 & 0 & \sin kl \end{vmatrix} = 0$$

$$kl\cos kl - \sin kl = 0$$

$$kl\cos kl = \sin kl$$

$$kl = \tan kl = 4.493$$

$$k^2 = (4.493)^2 = 2\pi^2$$

$$k^2 = \frac{2\pi^2}{l^2}$$

$$\frac{F}{EI - F(e_0 a)^2} = \frac{2\pi^2}{l^2}$$

$$Fl^2 = 2\pi^2 EI - 2\pi^2 F(e_0 a)^2$$

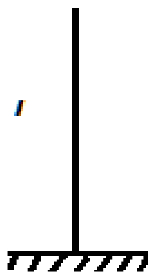
$$F\left[1 + \frac{2\pi^2}{l^2}(e_0 a)^2\right] = \frac{2\pi^2 EI}{l^2}$$

$$F_{cr} = \frac{\frac{2\pi^2 EI}{l^2}}{1 + \frac{2\pi^2}{l^2}(e_0 a)^2}$$

For smaller buckling loads, (n =1).The critical buckling load for column is given by,

$$\boldsymbol{F_{cr}} = \frac{\frac{2\pi^2 EI}{l^2}}{1 + \frac{2\pi^2}{l^2}(e_0 a)^2}$$

3.3 Cantilever end Conditions



$$\boldsymbol{v(x) = A + Bx + C\cos kx + D\sin kx}$$

$$\boldsymbol{v'(x) = B - Ck\sin kx + Dk\cos kx}$$

$$\boldsymbol{v''(x) = -Ck^2\cos kx - Dk^2\sin kx}$$

$$\boldsymbol{v'''(x) = Ck^3\sin kx - Dk^3\cos kx}$$

$$\boldsymbol{v(l) = A + Bl + C\cos kl + D\sin kl}$$

$$\boldsymbol{v''(l) = -Ck^2\cos kl - Dk^2\sin kl}$$

The boundary conditions are:

(i) $v(0) = 0$

(ii) $v'(0) = 0$

(iii) $v''(l) = 0$

(iv) $v'''(0) = 0$

$$\begin{bmatrix} v(0) \\ v'(0) \\ v''(l) \\ v'''(0) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & k \\ 0 & 0 & \cos kl & \sin kl \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

It is an Eigen value problem.

ijates
ISSN 2348 - 7550

$$\begin{vmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & k \\ 0 & 0 & \cos kl & \sin kl \\ 0 & 1 & 0 & 0 \end{vmatrix} = 0$$

$$1\begin{vmatrix} 1 & 0 & k \\ 0 & \cos kl & \sin kl \\ 1 & 0 & 0 \end{vmatrix} + 1\begin{vmatrix} 0 & 1 & k \\ 0 & 0 & \sin kl \\ 0 & 1 & 0 \end{vmatrix} = 0$$

$$-k\cos kl = 0$$

$$\cos kl = 0\ ;\ kl = \frac{n\pi}{2}\ ;k^2l^2 = \frac{n^2\pi^2}{4}$$

$$k^2 = \frac{n^2\pi^2}{4l^2}$$

$$\frac{F}{EI - F(e_0a)^2} = \frac{n^2\pi^2}{4l^2}$$

$$4Fl^2 = n^2\pi^2EI - n^2\pi^2F(e_0a)^2$$

$$F\left[1 + \frac{n^2\pi^2}{4l^2}(e_0a)^2\right] = \frac{n^2\pi^2EI}{4l^2}$$

$$F_{cr} = \frac{\frac{n^2\pi^2EI}{4l^2}}{1 + \frac{n^2\pi^2}{4l^2}(e_0a)^2}$$ For smaller buckling loads, (n =1).The critical buckling load for column is given by,

$$\boldsymbol{F_{cr}} = \frac{\frac{\pi^2EI}{4l^2}}{1 + \frac{\pi^2}{4l^2}(e_0a)^2}$$

IV. RESULTS AND DISCUSSIONS

Analysis of buckling loads has been done for nanobars of various spans with different nonlocal parameters. The results are tabulated and graphically depicted for various boundary conditions**.**

ijates
ISSN 2348 - 7550

4.1 Simply Supported Column

Table 1: Small Scale Effect on Simply Supported end Nanobars at Different Scale Coefficients for Buckling Loads.

Length in nm	Buckling Load			
	eo = 0	eo = 0.33	eo = 0.67	eo =1
5	19.384	9.3475	3.57245	1.78496
6	10.5	6.6	2.8	1.5
10	4.846	3.82048	2.2	1.2
15	2.15378	1.92422	1.44377	1.02781
20	1.2115	1.13531	0.948988	0.749586
25	0.7753	0.743431	0.658738	0.556059
30	0.53844	0.52285	0.479494	0.422681
35	0.395592	0.387109	0.36282	0.329326
40	0.302875	0.297878	0.283284	0.262444
45	0.239309	0.236178	0.22691	0.21334
50	0.19384	0.191781	0.185624	0.176443
55	0.160198	0.158789	0.154545	0.148128
60	0.134611	0.133615	0.130597	0.125985

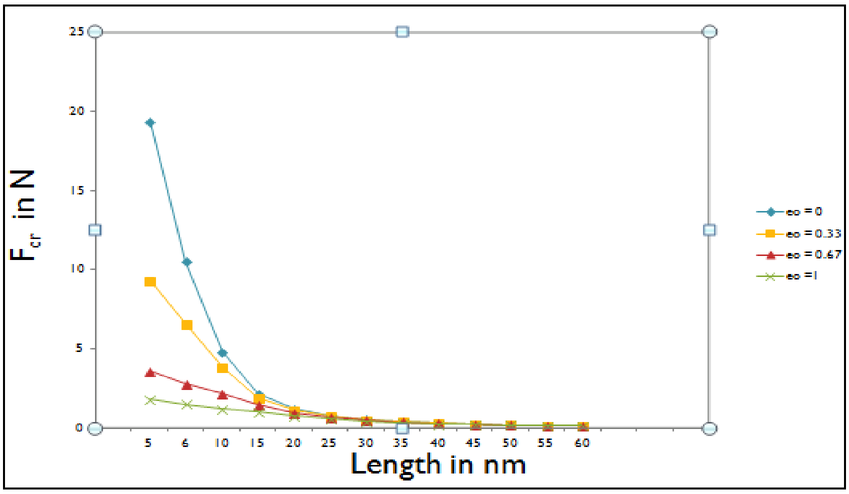


Fig .2 Small Scale Effect on Simply Supported end Nanobars at Different Scale Coefficients for Buckling Loads.

4.2 Clamped- Clamped Column

Table 2: Small scale effect on clamped-clamped end nanobars at different scale coefficients for buckling loads.

ijates
ISSN 2348 - 7550

Length in nm	Buckling Load			
	eo = 0	eo = 0.33	eo = 0.67	eo = 1
5	77.535	14.6316	4.141	1.9155
6	53.844	13.509	4.046	1.894
7	39.55	12.387	3.939	1.8711
8	30.287	11.303	3.822	1.8444
9	23.93	10.284	3.6988	1.815
10	19.383	9.34254	3.569	1.7833
15	8.615	5.8301	2.9016	1.5994
20	4.8459	3.8196	2.2993	1.3976
25	3.1014	2.6463	1.8149	1.2025
30	2.15377	1.924	1.4433	1.0273
35	1.5823	1.4547	1.1621	0.87632
40	1.21149	1.1352	0.94877	0.7492
45	0.9572	0.9089	0.78539	0.64356
50	0.775356	0.74339	0.65863	0.5559
55	0.64079	0.6188	0.55893	0.48315
60	0.53844	0.52283	0.47944	0.42259

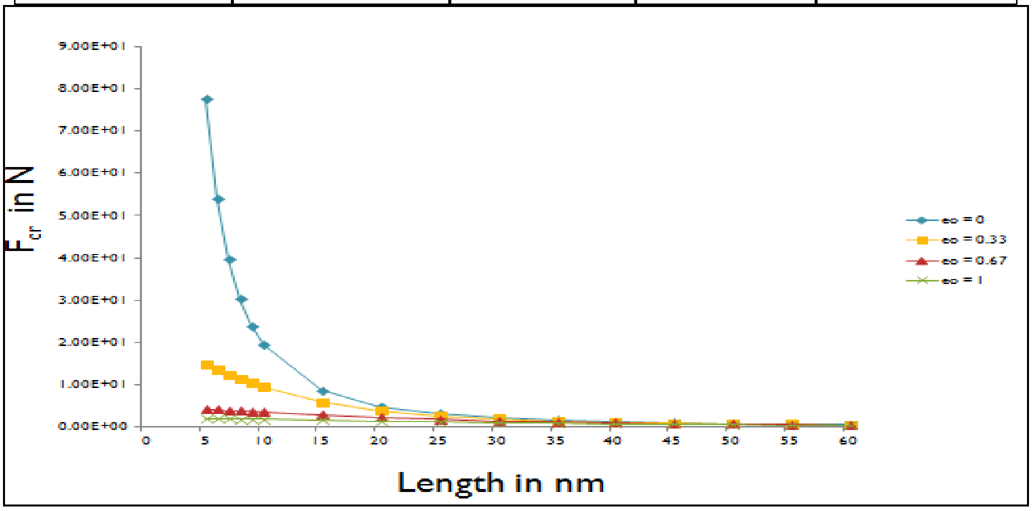


Fig. 3 Small Scale Effect on Clamped-Clamped end Nanobars at Different Scale Coefficients for Buckling Loads.

4.3 Clamped- Hinged Column

Table 3: Small Scale Effect on Clamped- Hinged end Nanobars at Different Scale Coefficients for Buckling Loads.

Length in nm	Buckling Load			
	eo = 0	eo = 0.33	eo = 0.67	eo = 1
0.000000005	3.87676E-08	1.2309E-08	3.93E-09	1.869E-09
0.00000001	9.6919E-09	6.3041E-09	3.01E-09	1.633E-09
0.000000015	4.30751E-09	3.477E-09	2.17E-09	1.349E-09
0.00000002	2.42298E-09	2.136E-09	1.56E-09	1.085E-09
0.000000025	1.5507E-09	1.4279E-09	1.14E-09	8.665E-10
0.00000003	1.07688E-09	1.0162E-09	8.64E-10	6.955E-10
0.000000035	7.91176E-10	7.5793E-10	6.7E-10	5.64E-10
0.00000004	6.05744E-10	5.8606E-10	5.32E-10	4.63E-10
0.000000045	4.78612E-10	4.6624E-10	4.31E-10	3.848E-10
0.00000005	3.87676E-10	3.7952E-10	3.56E-10	3.238E-10
0.000000055	3.20393E-10	3.148E-10	2.99E-10	2.755E-10
0.00000006	2.69219E-10	2.6526E-10	2.54E-10	2.368E-10

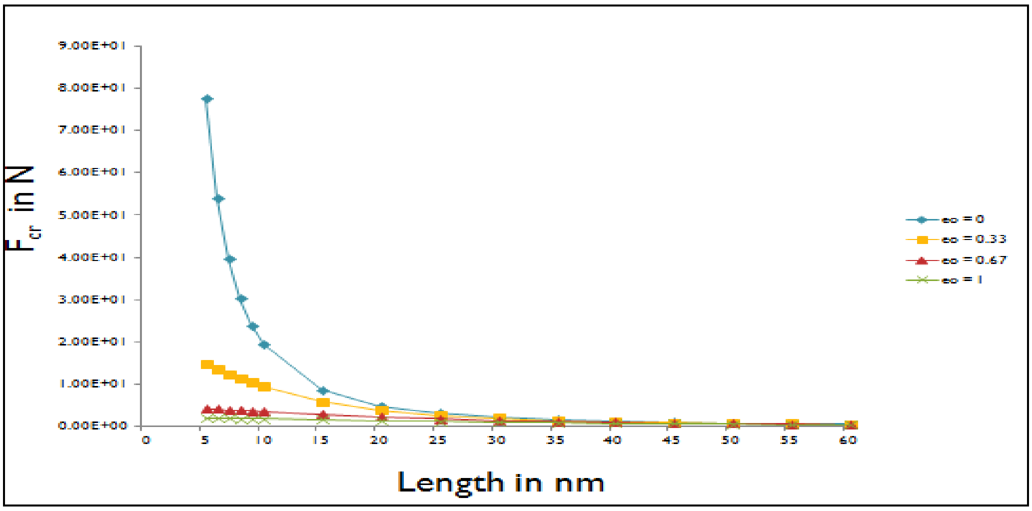


Fig .4 Small Scale Effect on Clamped-Hinged end Nanobars at Different Scale Coefficients for Buckling Loads.

4.4 For Cantilever Column

Table 4: Small Scale Effect on Cantilever Nanobars at Different Scale Coefficients for Buckling Loads.

Length in nm	Buckling Load			
	eo = 0	eo = 0.33	eo = 0.67	eo = 1
5	5	5	5	5
6	3.365	2.836	1.9022	1.24
7	2.472	2.174	1.579	1.094
8	1.893	1.713	1.3213	0.9639
10	1.21149	1.13523	0.94877	0.74929
15	0.53844	0.52283	0.47943	0.42259
20	0.302873	0.29287	0.283264	0.26241
25	0.193839	0.19177	0.185615	0.17643
30	0.1346	0.13361	0.130592	0.12598
35	0.098897	0.098358	0.096711	0.094156
40	0.075718	0.0754017	0.07443	0.072907
45	0.059827	0.059629	0.059019	0.058058
50	0.04846	0.048329	0.047929	0.047293
55	0.040049	0.0399606	0.039686	0.039249
60	0.033653	0.0335899	0.033396	0.033086

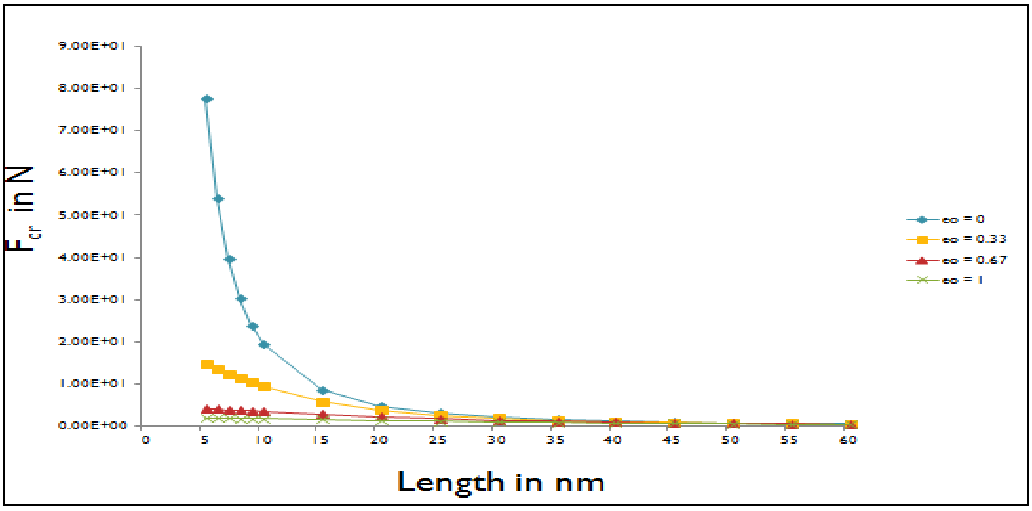


Fig .5 Small scale effect on cantilever nanobars at different scale coefficients for buckling loads.

4.5 Discussions

The effect of buckling loads is discussed for various boundary conditions for different scale coefficients and length to illustrate the influence of small length scale on the buckling of nanorods. Variation of buckling load parameter with length of rod is given for different scale coefficients e_0a for four boundary conditions considered in Figs. 1, 2, 3 and 4. It is obvious that, nonlocal solution of the buckling load is smaller than the classical (local) solution due to the effect of small length scale. Furthermore, increasing the nonlocal parameter decreases the buckling load. The result may be interpreted as increasing the nonlocal parameter for fixed L leads to a decrease in the stiffness of structure. Approximately, for $L \geq 20$nm all results converge to the classical buckling load. It means that the nonlocal effects are lost after a certain length. The nonlocal effects are more pronounced for C–C boundary conditions when compared with C–F boundary conditions. $e_0a = 0$ corresponds to classical solution. In general, the effect of nonlocal parameters is to reduce the buckling loads.

V. CONCLUSIONS

The above analytical investigations lead to the conclusion that the effect of the increase in nonlocal parameter is to decrease the buckling loads in nanobars. The nonlocal effects are very minimal at L=10 nm and disappear for greater lengths. The following are noteworthy:

(i) In the case of nanobars under axial forces, the nonlocal solution of buckling load is smaller than the classical elasticity solution..

ii) The buckling load of nanobars decreases with increase in nonlocal parameter.

iii) The buckling load of nanobars decreases with increase in length of the bar.

VI. ACKNOWLEDGEMENTS

The authors are thankful to the Management of Karunya University for providing necessary facilities to carry out the work reported in this paper.

REFERENCES

[1] Iijima.S., Helical microtubules of graphitic carbon, Nature, 354, 1991, 56-58.

[2] C.M.Wang et al., Recent Studies on Buckling of Carbon Nanotubes, Journal of applied mechanics reviews, 2010, Vol. 63.

[3] A. Cemal Eringen, On differential equations of nonlocal elasticity and solutions of screw dislocation and surface waves, Journal of applied physics 54, 4703 (1983).

[4] George R. Buchanan, Richard P. McNitt, Application of nonlocal continuum models to nanotechnology, International Journal of Engineering Science 41 (2003) 305–312.

[5] Q. Wang, K. M. Liew,X. Q. Hew, and Y. Xiang, Local Buckling of carbon nanotubes under bending, Applied Physics Letters 91, 2007.

[6] A. Sears and R. C. Batra, Buckling of multiwalled carbon nanotubes under axial compression, Physical Review B 73,(2006)

[7] Metin Aydogdu, A general nonlocal beam theory: its application to nanobeam bending, buckling and vibration, Journal of physica E 41, 2009.

[8] Helong Wu, Free Vibration and Buckling Analysis of Sandwich Beams with Functionally Graded Carbon Nanotube-Reinforced Composite Face Sheets, International Journal of Structural Stability and Dynamics Vol. 15,2015, No. 7.

[9] Y. Lei, T. Murmu, S. Adhikari , M.I. Friswell , European Journal of Mechanics A/Solids 42 (2013).

[10] Devesh Kumar,christian Heinrich,anthony M.Wass, Buckling analysis of carbon nanotubes modelled using nonlocal continuum theories, Journal of applied physics, 2008.

.