SOME RESULTS ON WEAK CO-MULTIPLICATION MODULES

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ABSTRACT

Let R be a commutative ring with identity and all modules to be treated as unitary modules. In this paper we obtain some results on weak co-multiplication modules.

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I. INTRODUCTION

Multiplication module was introduced by Barnard [8] in 1981. The dual notion of multiplication module as co-multiplication module was introduced by Ansari- Toroghy and Farshadifar [6] in 2007. Using the concept prime sub-module of module, the concept of weak multiplication module was developed and many more results have been given by Azizi Shiraz [4]. In the year 2009 the dual notion of weak multiplication module as weak co-multiplication module was introduced by Atani and Atani [5]. Some results on co-multiplication module were given by Saeed Rajaee [1]. This paper continues this line of research for weak co-multiplication modules.

Throughout this paper all rings will be commutative with non-zero identity and all modules will be unitary. If N and K are submodules of R-module M then the residual ideal N by K is defined as $(N :_R K) = \{ r \in R : r K \subseteq N\}$. Let N be submodule of M and I be an ideal of R the residual submodule N by I is defined as $(N :_M I) = \{ m \in M : mI \subseteq N\}$.

In the special case in which N= 0 the ideal $(0 :_R K)$ is called annihilator of K and it is denoted by $Ann_R (K)$ also the submodule $(0 :_M I)$ is called the annihilator of I in M and it is denoted by $Ann_M (I)$. A proper submodule N of an R-module M is said to be prime submodule of module M if $ra \in N$ for $r \in R$ and $a \in M$ then either $a \in N$ or $rM \subseteq N$[10] (also see examples in [11], [12].) The set of all prime submodules in an R-module M is denoted by Spec (M).

The aim of this paper is to investigate some results on weak co-multiplication modules.

II. PRELIMINARIES

In this section we give some basic definitions which will be helpful to understand the further results.

Definition 2.1 [8] An R -module M is said to be a multiplication module if for every submodule N of M, there exist an ideal I of R such that $N = I M$.

Definition 2.2 [6] An R -module M is said to be co-multiplication module if for every submodule N of M there exist an ideal I of R such that $N = (0 :_M I)$. It also follows that M is a co-multiplication module if and only if $N = (0 :_M Ann_R(N))$ for every submodule N of M .

Definition 2.3 [4] An R-module M is called weak multiplication module if M doesn't have any prime submodule or every prime submodule N of M, we have N = I M , where I is an ideal of R.

One can easily show that if an R-module M is a weak multiplication module then $N = (N :_R M)M$ for every prime submodule N of M [9].

Definition 2.4 [5] Let R be a commutative ring. An R-module M is defined to be a weak co-multiplication module if Spec (M) = ϕ or for every prime submodule N of M , $N = (0 :_M I) = Ann_M (I)$ for some ideal I of R. Also M is a weak co-multiplication module if and only if $N = [0 :_M Ann_R (N)]$ for every prime submodule N of M. We denote this concept by $N \subseteq_{WC} M$.

Definition 2.5[2] A submodule N of an R-module M is said to be pure submodule if $IN = N \cap IM$, for every ideal I of R.

Definition 2.6 [3] A submodule N of an R-module M is said to be co-pure submodule if $(N :_M I) = N + (0 :_M I)$, for every ideal I of R.

Definition 2.7[1] An R-module M is said to be fully pure (respectively fully co-pure) if every submodule of M is pure (respectively co-pure).

Definition 2.8 [7] If R is a ring and M is an R-module then M is said to be semisimple module if every submodule of M is a direct summand of M.

III. MAIN RESULTS

In this section we obtain some results on weak co-multiplication modules

Proposition 3.1

Let M be an R-module and $N \subseteq L \subseteq M$ then L/N is a weak co-multiplication submodule of M/N if and only if there exists an ideal I of R such that $L = [N :_M I]$. Let M be a semisimple R-module then M is fully co-pure and $L/N \cong Ann_M (I) / Ann_N (I)$.

Proof:

Since $IN \subseteq N$ for every ideal I of R, hence $N \subseteq L = [N :_M I]$. We consider M/N as an R-module. If L/N is a weak co-multiplication module then Spec (M/N) = ϕ or for every prime submodule $L/N \subseteq_{WC} M/N$ then there exists an ideal I of R such that

$L/N = [N :_{M/N} I] = \{ m + N \in M/N \mid I(m + N) = Im + N = N\} = \{m + N \in M/N \mid Im \subseteq N\} = \{ m + N \in M/N \mid m \in [N :_M I] \} = [N :_M I] / N$. Therefore $L = [N :_M I]$. The converse is clearly true.

Further let N be co-pure then $[N :_M I] = N + [0 :_M I]$. So

$L/N = N + [0 :_M I] / N \cong [0 :_M I] / N \cap [0 :_M I] = [0 :_M I]./ [0 :_N I] = Ann_M (I)/ Ann_N (I)$.

In particular let M be a semisimple R-module then there exists $K \subseteq M$ such that $M = N \oplus K$. Therefore $[N :_M I] = [N :_K I] + [N :_N I] = [0 :_K I] + N \subseteq [0 :_M I] + N$. Conversely it is clear that $[0 :_M I] + N \subseteq [N :_M I]$. Therefore $[N :_M I] = [0 :_M I] + N$ and hence N is co-pure.

Corollary 3.2

Let M be an R-module and $N \subseteq L \subseteq M$. If $N \subseteq_{WC} M$ and M/N be weak co-multiplication R-module then $L \subseteq_{WC} M$.

Proof:

We suppose that $N = [\, 0 :_M J]$ for some ideal J of R. Since $L/N \subseteq_{WC} M/N$ (by above Proposition 3.1), we have $L = [\, N :_M I]$ for some ideal I of R. So $L = [\, [\, 0 :_M J] :_M I]$. Therefore $L = [\, 0 :_M IJ]$ and hence $L \subseteq_{WC} M$. This completes the proof.

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REFERENCES

[1] Saeed, Rajaee, Some results on co-multiplication modules, International Journal of Algebra, Vol. 7 , No. 7, 337-340, 2013.

[2] Anderson, W. and Fuller, K. R. , Rings and Categories of Modules, Springer-Verlag, New York-Heidelberg-Berlin, 1974.

[3] Ansari-Toroghy, H. and Farshadifar, F., Strong co-multiplication module, CMU J. Nat. Sci., 8 , No. 1, 105-113, 2009.

[4] Azizi Shiraz, A., Weak multiplication modules, Czechoslovak Math J , 53,128, 529-534, 2003.

[5] Atani, Reza Ebrahimi and Atani , Shahabaddin Ebrahimi, Weak co-multiplication module over a pullback of commutative local Dedekind domains, Algebra and Discrete Mathematics, No.1, 1-13, 2009.

[6] Ansari-Toroghy, H. and Farshadifar, F., The dual notion of multiplication modules, J. math. 11 No. 4, 1189-1201, 2007.

[7] Osborne, M. Scott, Basic Homological Algebra, GTM-196, Springer-Verlag, New York, Inc., 2000.

[8] Barnard, A., Multiplication modules, J. Algebra 71, No. 1, 174-178, 1981.

[9] Saymeh, S.Abu: On dimension of finitely generated modules, Comm. Algebra, 23, 1131-1144, 1995.

[10] Gaur, Atul , Maloo, Alok Kumar and Prakash, Anand, Prime submodules in multiplication modules, International Journal of Algebra, Vol 1 , No 8 , 375-380.,2007.

[11] Lu, C. P., Prime submodules of modules, Comment. Math. Univ. St. Paul, 33, 61-69, 1984.

[12] Mc-Casland, R. L. and Moore, M. E., Prime submodules Comm Algebra, 20, 1803-1817, 1992.